Mathematics

1. Introduction

School mathematics is often viewed by students and teachers as application of procedures and formulae. This perception receives support from the nature of assessment followed in most schools. Frequently, tests are designed only to assess a student's knowledge of facts, formulae and procedure. As a result assessment of understanding of concepts and principles is given less importance.

Almost every teacher, when asked, justifies formal assessment as a way of finding out what the students have learnt. In this section on assessment in primary school mathematics, the emphasis is on assisting the teacher on how to assess the understanding of the learners, provide feedback to them as well as guage the effectiveness of her pedagogical practices in mathematics.

We have tried, in this part of the source book, to bring out the various dimensions that need to be assessed in order to give teachers a holistic view of the child's learning of mathematics as well as her own pedagogy and the curriculum. We have also tried to suggest the varied possibilities in terms of design of items and techniques for assessment that could be used by the teacher in both formal and informal settings.

For a thorough understanding of issues related to curriculum organisation, pedagogy and assessment in mathematics, teachers or need to refer to the NCF-2005, Position Paper of the Focus Group on Mathematics and the NCERT's mathematics syllabus for elementary stage.

2. Our approach to learning mathematics

The aim of school mathematics in the primary school years is to develop both 'useful' capabilities in the areas of numbers, number operations, measurement, spatial thinking and data handling. Further, it aims to develop the ability to reason mathematically, formulate and solve problems, make estimations, find approximate solutions and desirable attitude towards mathematics. As a result the child learns to communicate precisely through mathematical concepts and symbols. Visualisation and representation skills are also important and modelling situations using quantities, shapes and forms are part of mathematical development.

Children develop mathematical thinking through everyday interactions in the world. In fact the everyday world of most rural children is rich in oral mathematical traditions including techniques for computation, mathematics in riddles and recreation and even problem solving and optimisation. These are the cognitive resources that children already have access to, and which can be drawn upon by the pedagogic processes at school. School experiences introduce children to more formal aspects of mathematics beginning with the written representation system, formal understanding of number and number operations and then proceeding to many new concepts, operations and abstractions. It is well established that children's initial understanding of mathematics is 'concrete' and 'contextualised', and that learning is aided by designing tasks that involve the manipulation of concrete materials and provided in a context that enhances their meaningfulness and invites children to engage with them in a problem solving mode. It is through such activity that children 'construct' mathematical knowledge. Such a mode of doing mathematics can also be enjoyable and satisfying to most children. Mathematics is hierarchical and logically structured and children's learning is developmental. We can therefore expect gradual changes and deepening of children's understanding.

The attitude towards self as a learner of mathematics and towards mathematics itself are strongly formed by the nature of experiences children have while learning mathematics in turn influences their motivation and ability to learn. This includes not only the abilities of computation, mathematical reasoning and problem solving but also the appreciation of the beauty of mathematics.

2.1 Traditional approaches and suggested changes

Traditionally, assessment tends to follow the specific objectives of curricula, particularly as they are mentioned in the textbook. In this approach mathematical ideas and capabilities are broken down into items of knowledge to be taught, learnt and assessed. This lends itself to arriving at total marks in examinations and tests, but does not provide us with an overall comprehensive idea of where the child is, vis-a-vis the development of mathematical thinking and capabilities. Keeping in mind the objectives of learning mathematics and the dimensions of assessment, we present below the current practices and needed changes in assessment of learning mathematics.

Traditional Assessment	Suggested changes
Tends to be itemised towards atomised learning outcomes (specific learning objectives) and does not provide an over all comprehensive picture.	To be organised around broad dimensions along which mathematical knowledge would be conceptualised for the purpose of curriculum design and organisation of learning experiences.
Tends to avoid use of concrete materials and prefers paper pencil.	The child may be given only concrete activities to do and answers to report, and the teacher would use an observation sheet for notes.
Tends to avoid conversation and oral one-on-one discussion.	Tends to include conversation and one-on-one discussion with the child to explore her thinking.
Tends to be group administered only.	May be group administered also.
Tends to focus on computation.	Includes computation as one component of mathematical learning.
Tends to have questions with 'unique' solutions/or preferred method (as defined by the teacher) and avoid open ended exploratory type questions.	Allow for open ended exploration, problems with more than one right answer and also alteration in questions to permit teachers to interact with children's thinking.
Tends to emphasise memorised knowledge.	Recall of mathematical facts facilitates computation and mathematical reasoning, but children should also be able to 'reconstruct' facts and use them effectively as means to mathematically more important ends.
Does not permit any discrimination between rote application of algorithms and conceptual thinking/reasoning.	Designed to discriminate between the two.
Tends to view only the final answer of the child (product oriented) and to be judgmental.	Could be more process oriented. Ideally even while they are attempting paper-pencil tests, children should be observed while they solve the problems also, to see how they choose and alter strategies, how they respond to cues, etc.
Tends to be limited to knowledge of mathematics.	Would include metacognitive knowledge and ability to learn how to learn. Would include ability to

	express and understand mathematical language. Would also include attitude to mathematics, ability to persist with problems and problem solving.
Tends to assess based on right answers.	Uses erroneous answers in order to understand child's cognition.
Tends to be focused on single items of knowledge and are highly graded.	Could allow for variation in the task to elicit more than one level of response.

3. Our approach to assessment

The effort in this source book is to present tools for assessment that are consistent with the process of learning mathematics and to enable teachers to monitor the progress that children have made in learning mathematics.

The process of assessment in mathematics includes the following dimensions of mathematical learning:

- concepts and procedures
- mathematical reasoning
- dispositions towards mathematics
- using mathematical knowledge and techniques to solve problems
- communication

Assessment of children needs to consider their stage of intellectual development and to provide opportunities to work with mathematical concepts, procedures and problems, in contextually meaningful situations. This is not to say that the capacity for more abstract reasoning and symbolic manipulation is excluded, but this must certainly not be the only or exclusive method of assessment.

The developmental concerns also require that we provide ample opportunity for children to show their mathematical understanding independent of school-learning based symbolic representation. Hence we have included several tasks, which are oral, require one-on-one interaction, and involving the use of materials and pictures.

We have provided a range of items, but we have not tried to grade them while providing the examples. Often in mathematics, particularly number knowledge, gradation can be achieved by altering the size of numbers or by slightly increasing the complexity of the processing required. We have indicated some modifications of tasks for increasing complexity and difficulty within the descriptions provided. Teachers can compare and modify these illustrative examples for specific grade requirements, by referring to the syllabus for more specifications regarding grade-wise expected capabilities.

A great deal is known through research about the nature and development of mathematical concepts and procedures. It is possible and also desirable that through assessment teachers should explore the nature of children's learning of both concepts and procedures systematically, as this would enable teachers to plan their own instruction more effectively. We have identified about eight broad areas under which we have delineated concepts and procedures, which are learnt by children. The organisation of the later sections of this part of the source book follows this structure.

- Number (including number representation and decimal system)
- Number operations- addition, subtraction, multiplication and division

- Fractions
- Shapes and Spatial Thinking
- Measurement
- Problem Solving
- Patterns
- Data Handling

For each of these subsections we begin by presenting a discussion regarding what the learning of this area includes in terms of concepts, skills, procedural knowledge, thinking skills, vocabulary and argumentation. We also map its relationship to other conceptual areas both within mathematics and across the disciplines and in relation to life situation. We also present exemplar items that can be used for assessment and wherever possible we also include observations regarding interaction with children and interpreting the outcomes of observations. For the purposes of contrast some traditional assessment items along with strength and limitation are provided in boxes at the beginning.

The specific objectives approach of traditional assessment involves the detailed logical analysis of a mathematical concept. This is useful for the teacher to do, but while employing it, teachers need to keep in mind how various aspects of a concept complement each other to provide a sense of the larger concept being developed. Mathematical concepts are often hierarchically built up and for any concept there may be prerequisite ones. Analysis of such prerequisites can feed meaningfully into assessment to ensure that children are able to effectively demonstrate what they know rather than having their confidence destroyed by being assessed for things for which they lack the prerequisites. It is important not to confuse our analytical approach to assessment with the process of learning itself, which could be more unpredictable—we cannot be too sure how different things being learnt feed into each other and result in new learning.

In order to make an assessment of the development of a concept, skill or computational capability, a number of items need to be used. Variation in the assessment items is also essential as they provide children with wider opportunities to demonstrate their conceptual understanding. For example, if one wishes to assess children's conceptual development of number and addition, just asking a child '53+28' is not sufficient. More items on addition '53+28, 77+34, 13+39' may provide us with understanding that they are able to add these numbers or independent number facts. When a number of items are required they may be administered in some sequence, based on the response to the preceding item(s), and if need be a discussion with the child may be held. It is also possible to design tasks which require many capabilities of the child to be brought into play.

The items we have suggested in the source book related to a task may be used with some modification indeed, we expect teachers to construct additional items. The response to a particular item needs to be analysed to reinforce correct responses and to identify causes for wrong response and then to provide remedial instructions.

In order to assess children's learning, observation, oral questions and discussions, written tasks, projects and group work may be used. Assessment must not induce anxiety or frustration, and therefore it is necessary that all methods allow for children to experience success, show what they can do, and attempt challenging questions (in a problem solving situation), encouraging them to inquire, think, experiment, reason and solve.

Assessment would also need to include classroom observation in the form of mental notes the teacher makes as she teaches and while children work, regarding aspects of each child's participation as well as individual work characteristics. Some of the aspects to note would include: does the child ask questions, is he/she able to follow arguments and make their own, what does she/he do when confronted with a new type of problem, etc.

3.1 Materials

Several of the suggested items in this source book require different type of materials to be used. Some of the materials need to be made by the teacher.

Standard multi-purpose sets of materials collected and made by the teacher can be kept in the classrooms which can be used for various mathematical explorations and tests.

Some materials which are very useful in multiple ways include:

- Bag of counters/bottle caps/tamarind seeds and small baskets
- UTH sets: Unit, tens and hundred cubes and about 200 additional unit cube blocks

  Mala-moti sets, mala-moti cards
- Attribute set: standard polygonal shapes: triangle, square, rectangle, pentagon, hexagon, parallelogram in two sizes and in two/four colours
- Centimeter square grid sheet pads, and triangular grid sheet pads
- 1-100 number board
- Small cards/tokens with numerals from 0-200, and in multiple sets
- Flash cards or erasable cards and marker pens for writing numbers, small problems, shapes, names, for use in sorting based items
- Match-sticks in large numbers
- Jodo-gyan/Meccanno type lengths and joints for shape exploration
- 3-D shapes in various sizes
- Cloth pieces and cloth bags, string, pins, chalk etc.

3.2 Information from assessment

Assessment is essentially a process of gathering information. In the assessment of learning, if marks alone are used they do not tell us much about what is being learnt or how.

Even the child's statement 'I don't know' provides us is more valuable information, and we must not mark 'zero' for the question. In addition to telling us that the child does not know the answer, it does tell us that the child is confident and comfortable enough to say that she does not know.

In order to help the quality of learning, a record the following aspects may be kept child wise:

- Mathematical concepts (elaborated into various dimensions and subsections)
- Mathematical reasoning (able to follow an argument/able to provide an argument, justification, etc.)
- Attitude towards mathematics (persists at task, confident about ability to do, etc.)
- using mathematical knowledge and techniques to solve problems (can solve problems in more than one way if possible.)
- communication (initiates questions/ explains to peers, etc.)
- metacognition (able to explain how and why she did what she did and reflect on the procedures followed by others.)

The primary purpose of assessment information is to provide feedback to the child. This is crucial to enable children to develop their own confidence and to nurture emerging mathematical abilities.

Assessment of all the above mentioned dimensions will provide teachers with insights into the nature of individual attention to be provided during teaching learning process.

When assessment is taken across the whole class, the teacher could also gain insights into her own pedagogic practice help her identify areas where she needs to focus more or alter her current practices. It would also give her a sense of appropriateness of the overall level of the tasks/curriculum etc., which she could use in making modifications in what she is doing.

4. Number (including number representation)

Learning numbers basically has three inter-related but distinguishable aspects.

- **Number Sense**: This involves the ability to assess the size of a collection of objects/group, using estimation or counting. This is one of the most useful and basic skills in daily life and forms the basis of much of later learning.
- **Number representation**: This involves both naming and writing numbers. A deeper understanding of place value and the value of a numeral depending on its place in the number form the basis for more abstract understanding of number and representation.
- **Relationships between numbers**: It involves the ability to use number in ordering and sequencing, ‘before’, ‘after’, the ‘third after’ etc. are some of the expressions associated with ordering. This enables children to appreciate numbers as entities in themselves, to develop an understanding of the underlying logic and the ability to play around with numbers as abstract ideas.

The development of the concept of number in children involves the development of all these three areas. Generally, in school mathematics, number representation receives the maximum attention and effort instead of development of the number sense. These areas are developed in the school mathematics textbooks with suggestions to use concrete materials, followed by visual representations and eventually numerical manipulations. Some of the key aspects to look for in the development of number is whether the answer is being given from rote memory or whether there is some understanding also involved. This is often not easy to distinguish and more than one item may be necessary.

Standard items used to assess children's knowledge of number are usually questions of the type given below in paper-pencil form.

- write one to one hundred in numerals and in words.
- fill in the missing numerals: 23,---.---.---.---,28.

These items can be answered by a child who has grasped number concepts, as well as by a child who has only rote memorised knowledge. These items do not tell us much about the quality of the mathematical knowledge that children do have.

The three areas are interrelated which is being illustrated in subsequent sections. Many of the items provided below can be adapted to the developmental level of the child, on the basis of the size of numbers being dealt with.

4.1 Number sense

Number sense is the key conceptual part of understanding of numbers that needs to be assessed. This is also the part that is frequently missed out, as in traditional items it is difficult to say whether the child is answering from her knowledge of representation and counting or of number. It is only in unconventional number comparison problems and combining, decomposing problems and estimation problems asked in the oral or concrete mode that one is able to develop a sense of the child's number sense.

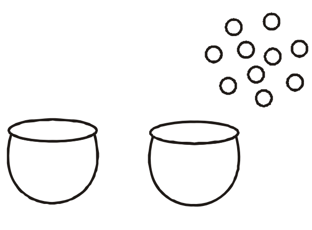
Especially in the beginning stages, the development of the naming of number and counting go together with the development of number sense. Although counting seems to be more 'social' and mechanical learning, we have included it in this section to bring out in addition the aspects of the development of counting that are dependent on developing number understanding, as well as using counting as a tool to deal with numbers. It is equally important for the teacher to try to understand the child's development vis-a-vis the skills involved in, say, counting and the naming of numbers.

Counting, naming of numbers, and writing numbers are some of the most basic skills that children begin with in number knowledge. Several of the assessment items provided below suggest the dimensions of what is to be assessed in this. In many Indian languages, the naming of numbers creates some difficulty for children. These are important to detect and note, but they tell us more about knowledge of representation and not about whether children have number concept. A final aspect worth mentioning is the importance of estimation of the size of a collection.

Item 1: (counting and counting techniques)

Material: 30 counters and two bowls.

Tell the child: 'Put 17 counters in a bowl'.



Note the strategy the child employs to put 17 counters in the bowl. DO NOT instruct her to count and put 17 in the bowl. Rather note whether she counts and if she does, then how she counts… in ones, or twos, or more. Depending on her confidence with the counting system, she will progress to more efficient ways of counting. Also note how she is minimising the possibility of errors in her counting/need to recount. Does she clearly put aside the counters she has counted? Does she arrange them in some order, say, in fives, or twos, to assist in tracking the counters counted and for recounting? She may make small errors, say for example, give you 16 or 18 instead of 17, but based on the total activity, you can decide how confident and knowledgeable she is with cardinality and the use of counting.

4.2 Number representation

Number representation includes the knowledge of number names and the symbolic representation of number in the form of numerals. Both oral and written representation of number is important, both reading and writing of number representation is important in the early years. However, writing the expanded version of number names such as 'five hundred and seventy one, five hundred and seventy two,…' are not mathematically enriching or relevant activities. Rather than seeing number representation as mechanical symbols to be learnt as rote, we can recognise that the names of numbers and representational issues equally use knowledge and abilities such as extending names to name new numbers, or extracting information regarding number size from the name of a number, involves the use of number knowledge. Place value is an important aspect of number representation and a core requirement of mathematics. The items suggested for place value help us examine some core ideas such as grouping, counting in ones, tens and hundreds.

Item 2: (recognition of numerals and knowledge of number names.)

Materials: number cards—set of cards upto a maximum of ten, with random numbers from the range of numbers you expect the child to know, including units, numbers with zero in the units or tens place and numbers where the digits are reversed e.g. 12, 21; 46, 64 etc. The range of numbers represented would be based on the expected range of the child's number knowledge. Some larger numbers may also be included to explore if children know/can extend their number knowledge range.

Children are shown one card at a time and asked 'what is this number?'

Note in particular if the child is confused about numbers where a zero is involved, and where the numerals are reversed. Also note if there is a number above which the naming of numbers is confused or unsure.

Item 3: (recognition of numerals and knowledge of number names.)

Materials: A number board with 0-9 in the uppermost row, and going on till 99 or one hundred (or more) (like a snakes and ladder board).

Point to the zero and ask the child to start reading the numbers.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33						
40									
50									
60									
70									
80									
90									

This item will enable you to understand if the child knows number names and the counting sequence. It will not establish clearly if the child is 'reading' the numerals she is seeing or if she is simply saying things from rote memory. Note if she can recognise and name 'zero'. If she cannot and hesitates, then place a card over the zero to cover it and ask her to read from 'one onwards'. See how far she is able to read.

If the child succeeds in the above part, then you can try this: point to a number midway, in the range the child is able to count and ask her to start counting from that number onwards.

This will enable you to judge if her earlier response was purely rote recitation or if she was reading the numbers with recognition.

You could also pick a number say '12', and then run your finger down the column to '22', '32' etc., and ask the child to name these numbers.

If the child has a good sense of place value and the structure of the number naming system, she will be able to name subsequent numbers with ease.

Item 4: (writing numerals)

Materials: number cards—set of cards upto a maximum of ten, with random numbers from the range of numbers you expect the child to know, including units, numbers with zero in the units or tens place and numbers where the digits are reversed e.g. 12, 21; 46, 64 etc. The range of numbers represented would be based on the expected range of the child's number knowledge. Some larger numbers may also be included to explore if children know/can extend their number knowledge range.

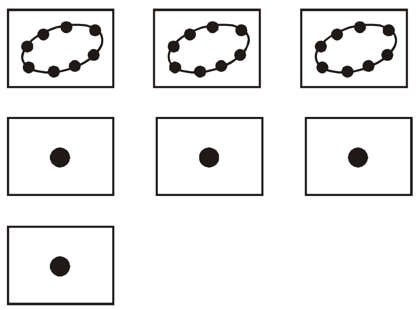
Pick out numbers beginning with smaller numbers with which the child may be more familiar, and then at random.

Read out the number on the card and ask child to write the numeral on the paper/slate provided.

Note in particular if the child is confused about numbers where a zero is involved, and where the numerals are reversed. Also note if there is a number above which the naming of numbers is confused or unsure.

Item 5: (Knowledge of grouping round tens an units (basis for decimal representation)

Materials: UTH cards with beads depicting sets of ten (five cards) and cards depicting one bead (five cards) OR UTH kit, with single cubes (five) and tens bars (five).



(To familiarise the child with the cards): Show all the ten bead cards to the child and tell: 'Each of these cards has 10 beads on it'. Show the single bead card and ask: 'How many beads are there in each of these? Put before the child one 10-bead card and ask: 'How much is this?' Pick up that and freshly lay two 10-bead cards and ask the same question.

(a) Pick up all the cards and out of them pick out one 10s and three ones and lay them out in front of the child. Ask: ‘How much is this?’

Pick up all the cards and out of them pick out three 10s and five ones and lay them out in front of the child. Ask : ‘How much is this?’

Pick up all the cards and out of them pick out two 10s and seven ones and lay them out in front of the child. Ask: ‘How much is this?’

Note the child’s response in each case. Also observe and note down whether the answer is provided immediately, whether she physically picks up or considers handles the tens and the ones and computes and announces the answer (also note if she proceeds with the tens first and then the units or the other way round), and also note if she counts all. This will indicate to you the familiarity and confidence the child has with UT and handling numbers in the form of units and tens.

(b) Give all the cards to the child ask her:

‘Represent 12 with these cards’

‘Represent 40’

‘Represent 23’

Note the child’s response in each case. Also observe and note whether the answer is provided by immediately laying out the cards working with tens and then with ones, or the other way round, and whether she counts all or counts in tens.

These items help to assess both readiness for and familiarity with the decimal place value system, by basically investigating if children are able to decompose and recompose number in tens and units. The same could be extended to 100s also.

Item 6: (Number Sense and representation).

Continue the number series

From 7 onwards to five more numbers, 7, _,_,_,_,_ .

From 28 onwards to five more numbers. 28,_,_,_,_,_.

Note in particular the child’s facility while going over from 9 to 10 and from 29 to 30. In case the child hesitates, you may want to prompt and scaffold so that the child is able to go over and continue the series. It is an indication that the child is aware and expects some change in the writing pattern, but is unsure about the change itself. More familiarity in the form of exposure and opportunities to count over is most likely what she needs.

Item 7: (Place value system)

Using UTH blocks represent 100, 137, 103 and 230

Item 8: (Place value system)

Name a 4 digit number which has 7 in the tens place and 5 in the thousands place

Item 9: (Place value system)

Using the digits 7,0,1,8 make the largest/smallest number using the digits only once each.

Item 10: (Number Representation on number line)

Tasks involving the number line could be used with children who do have some exposure and familiarity with the number line. Here we would be interested to know if children have developed a visualisation of numbers as positions on the number line and are able to also visualise relative positions of numbers on the line.

Draw a number line beginning with zero and continuing till beyond 50. Ask the child to point out some numbers for you on the line—5, 32, 43 etc. then ask her to show you 15.

'Which is closer to 15 – 32 or 41?'

' Which is closer to 15 – 11 or 19?'

'Which number is equidistant from 15 and 19?''

4.3 Number relationships

In many respects, number relationships is an extension and development of what we have described earlier as 'number sense', and could be considered another level of the same. It involves the development of conceptually more powerful and meaningful ways of thinking about numbers, their size, their relationship with each other, and their representation. The important aspect of this section though is that number is dealt with in a more 'abstract' manner, rather than being context bound. Naturally this area involves comparing numbers and also number operations, although the main difference is that the focus is on number conception and not so much on carrying out a number operation. Several aspects of this are also taken up in the later subsection called 'patterns'. We have not separately emphasised aspects of number learning such as 'odd-even', 'prime', 'factor', 'multiple' in this sourcebook. These properties are important in the more abstract conceptual development of number and arise out of seeing numbers in relation to each other, operating on them in particular ways, and abstracting and generalising common properties and features to describe a number. We take up such ideas here.

Item 11: (comparison of number representation)

Materials: number cards—set of cards upto a maximum of ten, with random numbers from the range of numbers you expect the child to know, including units, numbers with zero in the units or tens place and numbers where the digits are reversed e.g. 12, 21; 46, 64 etc.

Place all the cards out in front of the child and ask: 'Which is the biggest/smallest of these numbers?'

Note whether the child scans all the cards and picks out the biggest/smallest, also note if she chooses a card with a number which is bigger/smaller, but may not be the biggest/smallest. You could ask her to check (whether she is right or wrong) and note what strategy she uses to check her answer. You will want to note whether she compares numbers and reviews.

Place all the cards out in front of the child. Pull out one card (about the middle-most number) and place it above the set. Ask the child: ‘Put all those numbers which are smaller than this on this side (indicating the left) and all those numbers that are bigger than this on this side (indicating the right).

Note whether the child scans all the cards and picks out the biggest/smallest, also note if she chooses a card with a number which is bigger/smaller, but may not be the biggest/smallest. You could ask her to check (whether she is right or wrong) and note what strategy she uses to check her answer. You will want to note whether she compares number and reviews. It would be sufficient if the child gets this ‘mostly’ right. Note the errors: is it because she is mixing up, say ‘21 with 12’ (digit reversal) or she systematically misreads numerals where there is a zero?

Item 12: (relating cardinality and ordinality.)

Ask the child: ‘Tell me a number which is bigger/more than 11 and smaller/less than 20’.

Ask the child: ‘Tell me a number which is bigger/more than 110 and smaller/less than 120.’

Item 13: (recognising and continuing a pattern-continuing number series as opposed to continuing number sequence).

Continue the series 8,10,12, to five more numbers 8, 10, 12, ___,___,___,___,

This task is more advanced than a number sequence task, and can be used with children who have an established facility with the number sequence itself.

Continue the series.

13, 20, 27, …..

113, 120, 127,…..

100, 117, 137,…..

Item 14: (Place value system).

Given five numbers : 1007, 1070, 1700, 7100, 7100 arrange them from the biggest to the smallest.

4.4 Indicators

The indicators/markers of quality for assessment of achievement in development of numbers for Classes I-III are presented in the box. The teachers can develop their own indicators based on the syllabus.

		Numbers
Class I	:	Reads and writes numbers upto 20.
Class II	:	Reads and writes numbers upto 100.
Class III	:	Reads and writes numbers upto 1000.

5. Number Operations : Addition, Subtraction, Multiplication and Division

5.1 What to Assess and How

In this section we deal with four operations- addition, subtraction, multiplication and division.

In the development of arithmetic operations, the following two areas are examined

- Meaning of operations and their (mental) representation. The former relates more to understanding number sense, while the latter involves more aspects of number representation. The concepts of number and representation are in turn strengthened and made more powerful with the development of concepts of operations.
- Computational procedures including strategies for computation.

5.2 Meaning of Operations and their Representation

Word problems are important as they provide contexts in which the meaning of operations can be constructed. Problems can vary from each other in the complexity of the situation and which suggest different levels of difficulty. Assessment would need to help a teacher ascertain what kinds of word problems are within the child's cognitive grasp and where she may need the teacher's intervention to find solutions. Assessment would therefore address the following:

a. the child's ability to solve graded problems of different types – starting from the ones that are conceptually simple, to the difficult.

b. The child's representational capabilities—from concrete objects to numerical non-standard representations and finally standard numerical representation in terms of operations and conventional signs.

c. the child's ability to formulate problems of different types to fit in a given number sentence.

d. the efficacy and range of procedures used by the child to solve problems.

The following provides four main analytical categories for types of word problems in addition and subtraction. We have also indicated the difficulty level alongside.

I. Add to (join sets),

II. Combine (Part-Part-Whole problems)

III. Take From (partition of set),

IV. Compare.

In multiplication and division the five main types of problems are as follows:

	Type	Example
Multiplication	Multiplying factor	Chunni has three flowers and Munni has three times as many flowers. How many does Munni have?
	Rate	There are seven children and each one has two balls. How many balls are there in all?

	Cartesian Product	A dress is made in three different sizes and four different colours. How many dresses do I have from which to choose?
Division	Sharing	There are fifteen watermelons. Subba wants to arrange them in three equal rows. How many should he put in each row?
	Grouping	There are fifteen watermelons. Subba wants to arrange five in each row. How many rows can he make?

There is a tendency to restrict word problems when they are introduced to small number manipulations. As children become familiar with 3 or 4 digit numbers as well as with more abstract contexts, the same problem types would still need to be repeated. As can be seen from the illustrative examples, conceptually no difference is made between addition and subtraction and both operations may be introduced quite naturally together.

Type I : Add to : In 'add to' type of problems, something is being added.

Category	**Sub type**	**Sample problems and representations**
Add To	Result unknown	Rani had 5 pencils. Her father gave her 2 more. How many does she have now?
Add To	Change unknown	Rani had 5 pencils. Her father gave her some more. Now she has 7 pencils. How many pencils did her father give her?
Add To	Initial unknown	Rani had some pencils. Her father gave her 2 more. Now she has 7 pencils. How many pencils did she have to start with?

Type II: These are situations involving combining two group of things.

Combine	Whole unknown	Lata has 5 kites and Rama has 2 kites. How many do they have altogether?
Combine	Part unknown	Lata and Rama together have 7 kites. Lata has 5 kites. How many kites does Rama have?

Type III: Take from. In 'take from' type of problems something is being taken away.

Take From	Result unknown	Ali had 7 mangoes. He ate 5 of them. How many mangoes does he have now?
Take From	Change unknown	Ali had 7 mangoes. He ate some of them. Now he has 2 mangoes left. How many did he eat?
Take From	Initial unknown	Ali had some mangoes. He ate 5 of them. Now there are 2 mangoes left. How many mangoes did he have to start with?

Type IV: Here also two groups are involved and the numbers in each are being compared with those of the other in different ways.

Compare	Difference unknown	Ali has 7 kites. Rani has 5. How many more kites does Ali have than Rani? OR

		Ali has 7 kites. Rani has 5. How many less kites does Rani have than Ali?
Compare	Larger value unknown	Ali has 5 kites. Rani has 2 kites more than him. How many kites does Rani have?
Compare	Larger value unknown	Ali has 5 kites. He has 2 kites less than Rani. How many kites does Rani have?
Compare	Smaller value unknown	Ali has 7 kites. Rani has 2 kites less than him. How many kites does Rani have?
Compare	Smaller value unknown	Ali has 7 kites. He has 2 kites more than Rani. How many kites does Rani have?

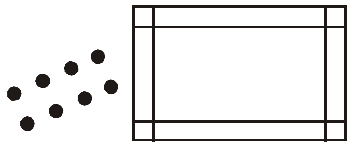
Item 1:

Material: Counters and two bowls.

Place the pre-arranged bowls before the child and say:

'There are 11 counters in this bowl and 5 in that bowl. In all how many counters are there?'

Item 2:



Present a tray to the child in which there are 15 counters visible and seven more are hidden under a handkerchief.

Ask her. 'In all there are 22 counters, how many are there under the handkerchief?'

Item 3:

Material: 30 counters and two bowls.

Ask the child to give you 17 counters.

Then ask her: 'How many more do you need to give me so that I have 20 counters with me?'

'How many more should you give me so that I will have 26 counters with me, in all?'

Note whether she is able to mentally compute and give you the answer. Note whether she is (a) picking up and using the counters in front of her (b) counting on (c) writing on a piece of paper and computing (d) some other strategy. This will again give you an indication of her confidence with concepts of number and cardinality.

Item 4:

a) ' There are 12 jasmine flowers in a basket and 7 jasmine flowers on a plate. Tell me how many flowers are there in all.'

b) 'In a village there is a girl called Suma. She had 17 mangoes with her. She gave 8 mangoes to the aunty who lives next door and used the rest to make chutney. How many mangoes did she use for the chutney?'

Note : Tell the child that he should solve this problem in his mind. But, if the child expresses his wish to write and solve it, he should be free to do so. Counter and paper pencil are near by. Two sample problems are given below. You can construct more sample problems on the patterns provided above.

If the child has not used paper pencil to solve the problem, after her answer you may also ask her: 'can you also solve it using paper and pencil?' and see how she represents the problem.

Item 5:

Make up your own problem for 3+7=

Make up your own problem for 7 + ? = 22.

Note : If the child is familiar with representations or if she uses her own form of representations and you are able to access it, you may provide illustrative number sentences and ask her to make a problem for this.

5.3 Computations

Computation in the basic operations involves both the development of informal strategies, ability to estimate, and also the more abstract understanding of the operation and the properties of the operation.

Children use strategies based on counting, such as 'counting all', 'take away' in order to process computational requirements. Formal procedures based on number representation using the decimal system are very efficient ways of adding or subtracting large numbers. Here children need to remember a number of rules regarding carrying over and borrowing. Children may also invent their own procedures to compute. In fact, children also learn to employ efficient ways of grouping of numbers to minimise errors. Assessment tasks thus needs to help teachers understand where the child is.

Additive Procedures		**Corresponding Subtractive Procedures**
Count all	The child physically puts the two part quantities together using objects or pictures and then counts the whole collection.	Take away
Add on up to	Similar to the above, except that the child counts out the first quantity and then continues to add on the second.	Separate to
Count on	The child does not explicitly count out the first quantity, but starts from that number or the next to count on by the second quantity. It can be noted here whether the child demonstrates an understanding of commutatively by choosing the more efficient way of computing – that is, choosing to start with 5 (the bigger number) and count on 2 (the smaller) rather than the other way around.	Count down
Count up to	This procedure, though additive in nature, is actually used for subtraction situations and demonstrates the child's understanding of the relation between addition and subtraction. In this, the child when faced with a subtraction problem of the type 7 – 2, counts up to 7 (the bigger number) - starting with 2 (the smaller number).	Count down to
Use known facts	A child at this level of thinking already knows some basic facts and uses those to arrive at the unknown sum. For instance, to find 8 + 3, the child might use 'up over tens', making use of 8 + 2 = 10, a fact known to her (8 + 3 = 8 + 2 + 1 = 10 + 1 = 11)	Use known facts

Item 6: (mental and symbolic representation of operations)

Match the items of group A with the statements of group B.

Group A	Group B
10 ÷ 2= ?	Ninety bottles had to be placed equally in two rows. How many bottles were there in each row?
10 x 2 = ?	Mother bought ten sweets and shared them between Munni and Chunni.
90 ÷ 2 = ?	Out of ninety bottles two bottles broke. How many were left?
90 – 2 = ?	Ten mothers came with two children each. How many children were there in all?

Item 7: (multiplication as rate)

Materials required – water, a container, a cup

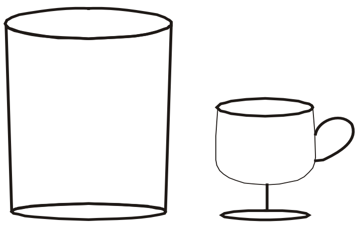
Ask each child to pour 2 cups of water into the container.

Then ask how many cups of water are there in the container.

When children give their answer, ask them how they found out.

Note the techniques children evolve to represent the problem to themselves and find the answer.

Repeat the same problem and this time ask children to pour three cups each, five cups each, etc. Note if they are noting any patterns and making any generalisations. Also note when they begin to 'find the answer' even before/without actually pouring water into the container.



Item 8: (Division as equal sharing)

Materials required: water, a container, six tumblers.

There are 6 glasses of water in the container and it has to be given to 3 children equally, how many glasses of water would each get? (Equal sharing)

At first the child can be asked to actually measure and find the answer.

Later as the problem is posed again and again, they could be prompted to generalise and answer verbally.

Item 9: (division as equal sharing/multiplication as rate)

Call 3 children and give them 15 pebbles. Ask them to divide it equally and see how much each gets —

Now give them a word problem that if they were given 18 pebbles how much would each get.

Convert the problem into a multiplication one by asking them that, if each wanted 5 pebbles how many pebbles should be there in all.

Item 10: (division as grouping)

The school children are going on a trip to the zoo. There are 120 children in the schools and they have to go by bus. Each bus can accomodate 30 children. How many buses are required? (Grouping)

Item 11: (multiplication as 'rate')

There are 6 biscuits in a packet. How many biscuits are there in 4 packets?

Each bar of chocolate has 8 pieces. How many pieces are there in 4 bars?

Item 12: (mental and symbolic representation of multiplication)

Make up a problem which corresponds to the multiplication, '16 × 5' in the context of shopping.

For example: Rita purchased 5 packets of pencils. Each packet has 16 pencils in it. How many pencils are in 5 packets?

This activity would help to assess the children's level of concept understanding.

Item 13: (mental and concrete representation of multiplication/division)

Place about 50 counters and provide child with loops of string/chalk and a large board to draw on.

'Show me the following with the materials. —

If 24 sweets are distributed equally in three groups then each group would get 8 sweets.

If 24 sweets are distributed in a group of 3 equally then 8 groups will be formed.

3 can be divided 8 times from number 24.

8 times 3 are 24.

3 times 8 are 24.'

5.4 Multiplication tables, patterns and properties

Multiplication tables tend to occupy a prime place in the learning of multiplication and division in schools. There is little doubt that knowing the tables is useful not only in computation but also later in developing analytical abilities to recognise and create number patterns, but the learning of tables need not be rote based and could be used to strengthen the conceptual development of multiplication. Further, children can be empowered by knowing how to make their own tables, to devise ways of extending the tables and also to use knowledge of facts to derive and deduce new knowledge. Thus there is need for the learning of the multiplication tables to shift from rote towards more creative and empowering experiences.

Item

If 4 × 8 is 32, what is 4 × 9?
If 4 × 14 is 56 what is 4 × 15?
If 13 × 241 is 3133, what is 13 × 242?

5.5 Indicators

The indicators/markers of quality for assessment of achievement in four operations for Classes I-V are presented in the box. The teachers can develop their own indicators based on the syllabus.

		Addition, Subtraction, Multiplication and Division
Class I	:	Adds and subtracts numbers upto 20
	:	Adds two single digit numbers mentally.
Class II	:	Adds and subtracts two digit numbers
	:	Adds and subtracts single digit numbers mentally
	:	Discusses situation involving repeated addition.
Class III	:	Computes sum and difference between two given numbers (do not exceeding 3 digit numbers)
	:	Constructs the multiplication tables of 2, 3, 4, 5, and 10.
Class IV	:	Divides a given number by another number.
Class V	:	Explains the meaning of factors and multiples.

It is presumed that the teachers formulate indicators corresponding the areas that are already introduced in the previous classes.

6. Fractions

<We could keep a cartoon on the following lines at the start of the chapter: A child raises his hand and answers correctly when the teacher asks the answer to 'Convert $19\ ^{17}/_{23}$ *into an improper fraction.' The teacher then asks the same child what whole number is 12/13 close to… the child is totally confused*>

Learning of fractions and decimal numbers are important for a primary school child because they expand her understanding of numbers to include quantities that are not whole. In the primary grades, students need to develop an understanding of fractions as a quantity, and as division or ratio.
Understanding of fractions at the primary school level can broadly be said to fall under three conceptual areas – 'part to whole or collection', 'division' and 'ratio'.

6.1 Conceptual Elaboration

A conceptual understanding of fractions for a primary school student would mean **understanding the multiple meaning of fractions**: This would involve a basic understanding of the nature of fractions – with a perspective on its many different meanings. This means is not limiting a child to the 'part of a whole' meaning of a fraction, but an introduction to other ideas – like fractions as part of a collection, fractions as a division operation or for comparison.

Many children's difficulties with mathematics start with the introduction of fractions. The reasons for that may be manifold. Many of the of whole numbers may get applied wrongly in the case of fractions – for instance, a child reasoning that ½ is less than 1/3 because 2 is less than 3. Also, compared to whole numbers, students spend very little time on the fundamental concepts of fractions, often moving quickly onto learning for operations and their properties. A third reason for student's difficulties – particularly in the case of common fractions – is the propensity that books and teachers have towards computations and other work with 'bigger' fractions, which have no bearing on the child's life. In fact hardly any of us would have ever done such calculations in real life or felt the need for them – except in the mathematics class! All these hurdles together hinder the child's understanding of fractional quantities.

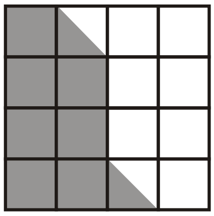
Keeping this in mind, it is desirable to keep the focus of the assessment of fractions on the fundamental concepts using commonly used fractions like ¼, ½, ¾ etc. and many algorithms students do not get ample time to it is recommended that at this level, conceptual work with commonly used fractions like ½, ¼, ¾ etc. should be focussed on, leaving algorithms for fraction operations and other mechanical work with 'large looking' fractions like 41/67 well alone. Developing the concept of fraction needs to be the key competency to be addressed at this level. Children need a variety of tasks to demonstrate their understanding of fractions.

6.2 Understanding the multiple meaning of fractions

Item 1

Type of Task: Performance task.
Material: Squared sheets.
Task description: Ask the students to draw many 4 by 4 squares, and shade half of each, in as many different ways they can think of.
The activity can be extended to have children draw different shapes on the squared sheet, and shade exactly half of each. A student should be able to justify why the part she has shaded is exactly half.

Sample Student Solution:

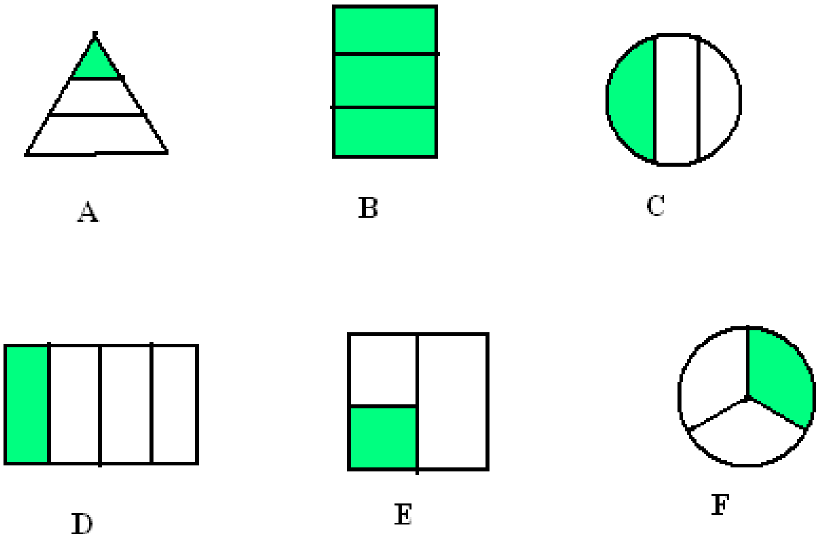


Shaded part is half because I have colored 7 full boxes and 2 half boxes while the same number of boxes are left.

The activity can also be extended to include common fractions – such as ¼ and ¾ also.
Note: In understanding fractions as part of a whole the constituent parts need to be equal. This item addresses that understanding. The first task also addresses the misconception that a square can be divided into half in a few set ways – mostly using vertical or horizontal lines.

Item 2

Type of Task: Written problem.
Problem: Which of the figures below are one third shaded?



Note: This item also addresses the understanding that constituent parts need to be equal. It also addresses several misconceptions – so for each figure, a student should be asked to justify why she says the shaded part is one third or not. Some students may show a flawed understanding of 'denominator' as total number of constituent parts, by counting the unshaded parts only and choosing figure D as being 1/3 shaded.

Item 3

Type of Task: Performance task.

Material: Collection of different objects – could be a vase with some flowers, for instance.

Task description: Suppose the vase contains 5 yellow flowers and 2 red flowers. Ask the students, 'What fraction of the flowers in the vase is red?'. The student is expected to say– 'Two seventh.' Some students may say 'Two fifth'. Such students lack an understanding of fractions as a part of a collection, in not taking into account all the objects in the set. Later, a collection of objects with 3 or more types can be used.

Note: Fraction as a part of a collection is an important understanding for students, since this is probably used more often in real life as we say, for example, "Almost two third of the class is absent today". It often happens because teachers focus more on the 'parts of a whole' model. As a result, various misconceptions can develop about this concept like a student saying 'Two fifth' instead of 'Two seventh' in the above example, without counting all the flowers in the vase.

Item 4

Type of Task: Performance task with oral questioning.

Material: Any material present in the classroom, which can be used for dividing or sharing – food, chart paper etc.

Task description: Give a child a chart paper and ask her to 'Give one fourth of this chart paper to her friends' and observe how she divides it. Does she make four exactly equal parts, or does she divide it approximately.

You can also make statements and ask them to analyse it mathematically. For instance 'Yesterday my friend shared a watermelon with me. He gave me the

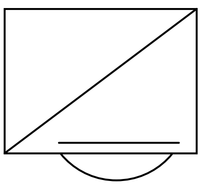
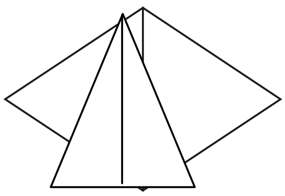
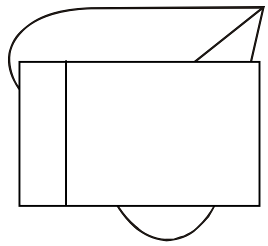
bigger half.' Ask them whether this statement is mathematically correct, if not what the error is, and give reasons.
Note: It is important to check for an understanding of fractions in context sometimes – otherwise students sometimes remember certain things only for problems and do not use it when actually needed.

Item 5

Type of Task: Performance task.
Material: A collection of objects – for instance toffees, crayons etc.
(It can be used as a colouring task instead of a real task)
Task description: Display a small set of objects from the collection – say 8 toffees. Ask a student to divide it into halves. If the child is successful, she can be asked to pick out a quarter of them, and then three quarters.
If a child can display understanding of such fractions, she can be given more challenging tasks like - to pick out ½ of 9, ¼ of 10 etc.
Note: This task checks to see the degree of comfort that individual children might have with commonly used fractions by asking them to pick out ½, ¼ etc. of a collection. Observe how the child attempts to do it. For instance, a child might already 'know' halves of some numbers – so he might straightaway take 4 toffees, Another child might have to actually divide physically into two equal parts, then recheck the parts are equal. Observe the procedures used for ¼ and ¾.

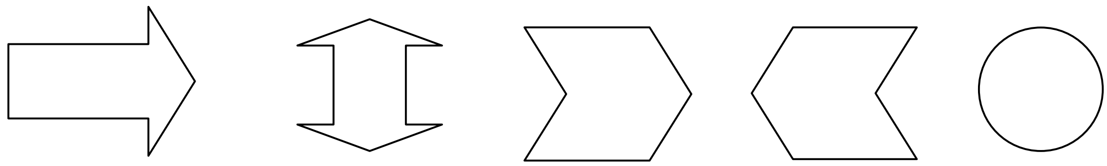
Item 6

Which of the following figures are divided into two equal parts?
(Have children understood what does equal half mean?)

 A	 B	 C
D	F	G

Item 7:

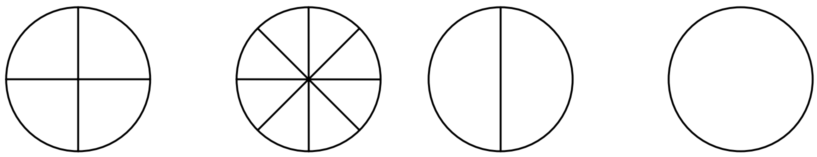
For the following figures, ask the child to divide the figure (you can ask the child to use colours to distinguish or just draw a separating line) into as many equal parts as possible.



This is meant to assess if the child has an understanding of the notion of ‘equal parts’ and that a single entity can be divided into more than 2 equal parts.

Item 8:

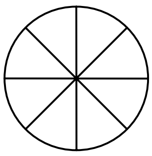
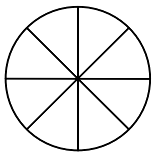
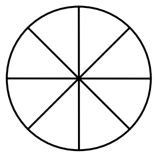
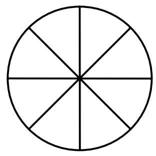
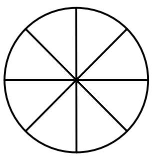
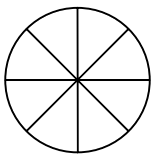
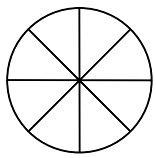
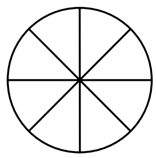
Ask the child to say how many parts each circle is divided into. In addition ask her to write the fractional value of each part of the circle (by shading) as compared to the whole ?
(Deliberately not shaded: *Concept of whole in the fraction*)



The part(s) of the circle is (are) not shaded in order to help the child to visually recognise the division of circles into equal parts and use appropriate numerals. Secondly we can also assess if the child is able to relate this to and represent it in the accepted mathematical symbolic language.

Item 9:

Shade the following as mentioned
(*Concept of whole in the fraction*)
or
Shade as required and ask child to write the appropriate fraction.

 Shade 3/8	 Shade 2/8	 Shade 5/8	 Shade 8/8
 Shade 1/8	 Shade 4/8	 Shade 7/8	 Shade 6/8

Item 10:

Type of Task: Written form

Problem: Write each of the following fractions in the boxes given below.

5/6 6/11 112/111 1/3 34/37 3½

Close to 1	Close to 1/2	More than 1

Note: Apart from checking for understanding of fractions numbers and estimating their value, this item helps in assessing understanding of fractions and the ability to estimate their values.

Item 11:

Type of Task: Written form.

Problem: Latha had half a paratha and Kartik also had half a paratha. But Latha said she had more paratha than Kartik. Can Latha be right? Give reasons and draw pictures if necessary to support your answer.

Note: The quality or otherwise of the two halves depends on which whole they are part of. This task checks for that understanding. Students who have understood this might reason that 'Yes, it is possible, because the paratha whose half Latha had might have been bigger than the one whose half Kartik had.' Other similar tasks can also be designed and used.

6.3 Identifying equivalent fractions

Item 12.

Type of Task: Performance task.

Material: Two equal pieces of chart paper.

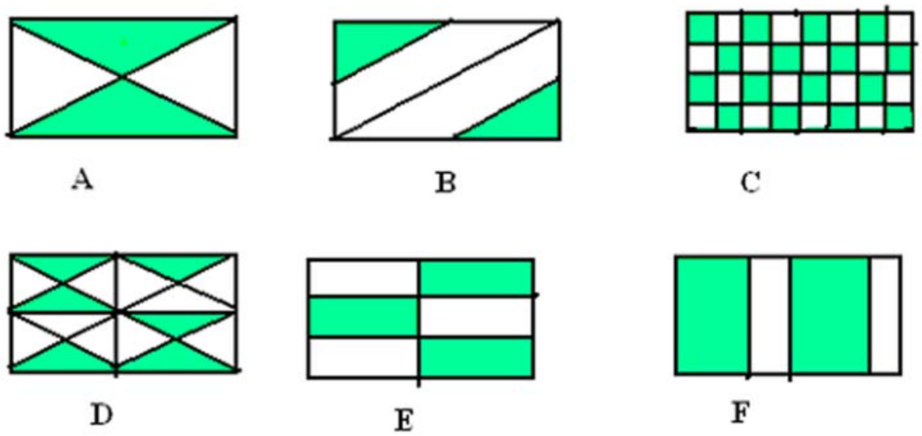
Task description: Tell the children "I will give Rani 2/3 of the first paper and Ali 4/6 of the second. Whose paper will be bigger? Why?"

Note: Encourage the children to try and predict with reasons, without physically dividing the papers. Observe how many can reason correctly that way. Later they can be allowed to actually divide the paper and check.

Item 13:

Type of Task: Written Problem.

Problem: Six equal rectangles are shaded in different ways. Which of them represent equivalent fractions? Give reasons.

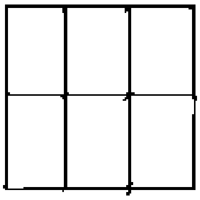


Note: Getting the students to reason about why a particular figure is equivalent to another is the important part of this task.

Item 14:

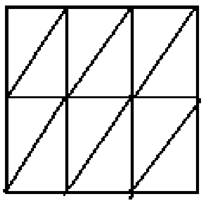
Type of Task: Written Problem.

Problem: Shade 2/3 of the square given below.



A

If a student finds figures like this (A) easy, slightly more challenging ones (like B) can be given.



B

Note: Using the idea of equivalence is implicit in the problem without stating it. If the child is successful in such tasks, it means she is looking at the parts in flexible ways and in terms of equivalence.

Item 15:

Type of Task: Written Problem.

Problem: Which is more: 1/9 or 1/10? Why? Explain in two different ways.

Later, the task might be extended - the students can be asked to reason whether 7/9 is greater or 7/10.

Note: The task is meant to give an opportunity for students to apply their understanding of equivalence. Asking for alternate ways of explaining also might be a good idea. Apart from the traditional 'writing as equivalent fractions' method, diverse student solutions should be encouraged and accounted for in the assessment. In case of fractions 1/9 and 1/10, if a student says, 'If the number of parts in which the whole thing is divided is more, the size of each part will decrease. So 1/10 is less than 1/9', he shows a good conceptual grasp of relation between fractions.

Item 16:

For each of the following, encircle larger of the two. In case they are equal write '=' between them.

a. $\frac{1}{6}$ $\frac{1}{8}$

b. $\frac{1}{7}$ $\frac{2}{7}$

c. 1 $\frac{4}{4}$

d. $\frac{3}{6}$ $\frac{1}{1}$

e. $\frac{4}{5}$ 1

This can be used to assess the ability of the child to compare two fractions.
Name some fractions equivalent to 8/12?
(How is the child is trying to find the answer? Is she multiplying 8/12 with 1, 2,… or trying few fractions such as 2/3 or any other fraction multiplied with 1, 2, 3…)
Observe the different approaches that the child takes to solve this problem.
Does she use an equal multiplier on the numerator and the denominator
What other strategies does she use?

Item 17:

Is 2/3 equivalent to 8/12?
(*Find out how the child is coming out with the conclusion?*)
Does ½ + ½ represent a whole? Give reasons.

6.4 Indicators

The indicators/markers of quality for assessment of achievement in fractions for Classes I-V are presented in the box. The teachers can develop their own indicators based on the syllabus.

	Fraction
Class IV	: Explains the meaning of $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{3}{4}$
	: Appreciates equivalent fractions of $\frac{1}{2}$ and $\frac{2}{3}$
Class V	: Compares fractions
	: Uses decimal fractions in the context of units of length and money

7. Shapes and Spatial Thinking

7.1 Conceptual Elaboration

Spatial reasoning skills and a working knowledge of geometry are necessary to understand, interpret and appreciate the world around. The development of this reasoning and knowledge is a part of cognitive development and finds expression in both kinaesthetic capabilities, cultural, artistic and technological aspects. School is concerned with the refinement and formalisation of such knowledge and reasoning, and developing concepts and language for expressing and conducting investigations of shape and space. These typically form the core of geometry taught in schools. The approach suggested for pedagogy and assessment, therefore, includes firstly the informal and intuitive concepts followed by the more formal aspects of shape.

Teaching and learning geometry at the primary level aims to develop in a child:

- Understanding shapes and their properties
- Visualisation and spatial reasoning skills
- Understanding fundamental geometric concepts
- Understanding important geometric ideas
- Developing logical reasoning abilities

These are discussed in more detail in the subsections dealing with the specific aspect.

> Traditional testing items tend to focus on identifying and naming standard two dimensional and three dimensional shapes and quickly move on to formal geometry of two dimensional shapes, usually involving the definitions of geometrical concepts.

7.2 Understanding Shapes and their Properties

Geometry begins with identifying and describing shapes. The primary classes are the ideal stage to help students refine and extend the rough, intuitive understanding of shapes that they bring with them as pre-schoolers. In the current practices, students first learn to name the basic 2-D and 3-D shapes, and then go on to learn the names of more and more complex shapes and terms related to them - sides, vertices etc. as they move onwards from class 1 to 5. However, it is very important to note that learning names and terms are merely stepping stones which will enable them to do more formal reasoning based work.

In order to further students' understanding of shapes and reasoning skills, it is important to do a lot of exploratory activities with both 3-D and 2-D shapes, getting them to examine similarities and differences and talk about their various attributes.

Specifically, students at the primary level should be able to recognise, compare, sort, build and draw two and three-dimensional shapes. Before learning to identify and name the common 3-D and 2-D shapes formally, students must be successful at informal tasks using shapes involving comparing, finding shapes like a given one, etc. Later, students can be asked to identify common shapes like square, circle, rectangle, triangle, cubes and cones. Initially, young children recognise a shape by its overall appearance, without looking at their properties. For instance, they may say that a given figure is a triangle because "it is pointed at the top." There are various gaps in the understanding of shapes for children functioning at this level. Some of the more common ones are

- Not recognising a shape on changing orientation. For instance, a child might recognise a triangle or square only in the 'regular' orientation – with the base horizontal and not in other orientations.
- Not recognising a shape if it is very small, very narrow etc.
- Thinking that only equilateral triangles are triangles.

In some cases these misconceptions might persist even after the child is in high school. While assessing, it is important to keep these in mind and design graded tasks.

Illustrative items:

Item 1: (Intuitive understanding of properties of shape:)

Two sets of different shapes (regular and irregular). One set is placed at some distance from the child (but within reach). In front of the child a small piece of cloth (large enough to cover each of the objects of second set) is placed and without allowing the child to see it, by turn each object is placed under the cloth.

Ask the child to feel the object through the cloth and to identify corresponding objects from the second set.

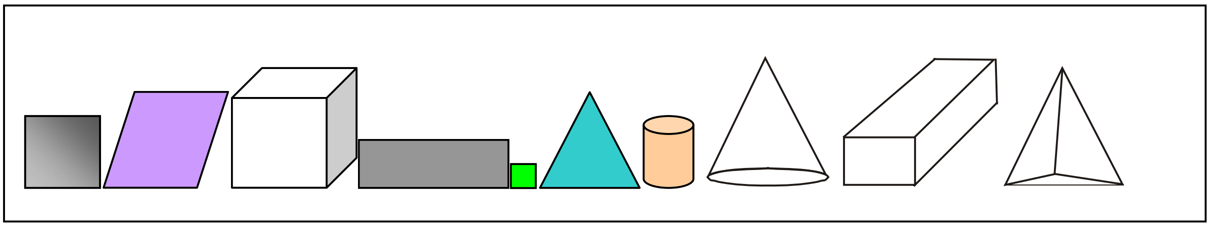
Notes: If the child knows the names of the objects, she may also name instead of pointing. Later, when the child is familiar with the properties and names of shapes, the same game can be played without keeping an array for the child as a reference. In addition, shapes that are fairly close to each other e.g. square and rectangle, may also be placed. Notice how the child feels for and assesses various features and properties, e.g. side, number of sides, vertices, equivalence of lengths etc.

Item 2:

Show a square cut-out to a child. Ask her to name the shape. If she names it correctly, rotate it around a little and ask what shape it is now. Note if she still identifies it as a square or not and ask her to give reasons for her answer.

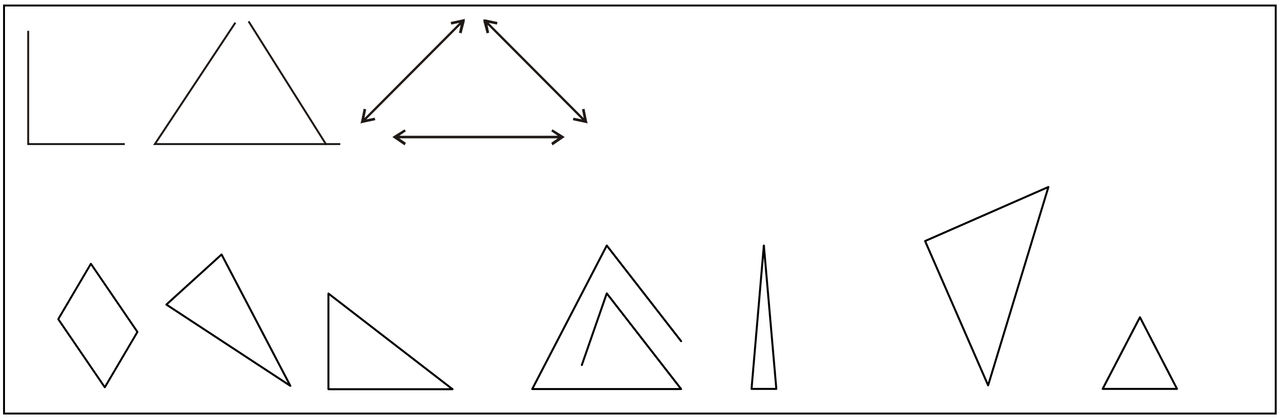
Item 3:

Give a collection of shapes to the student and ask her to sort them into 2 groups based on shape recognition (for instance, those that are squares and those that are not).



Item 4:

Which of the following are triangles and why?



Item Number: Identify, compare, and analyse attributes of two and three-dimensional shapes and develop vocabulary to describe the attributes.

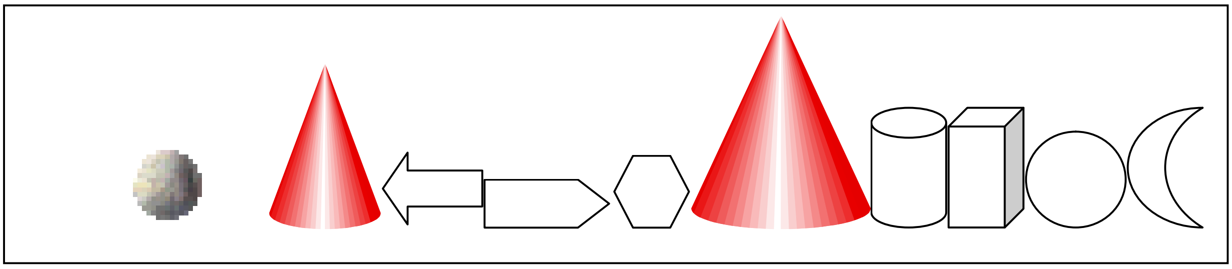
Item 5:

Ask students to collect different geometrical objects. Hold up one object at a time and ask them to describe it. You may demonstrate the task once. Note what words are used by students and whether they are able to describe objects using words such as point, vertex, sides, flat surface, curved, etc.

Item 6:

Give a collection of shapes (either only 3-D, or only 2-D, or mixed) to students and ask them to find those having a particular attribute. For instance:

i. Which of the following shapes have more than 3 vertices?
ii. Which of them at least one pair of sides equal?
iii. Which have a curved face?



Item 7:

Material: Matchsticks, meccano set or ice-cream sticks with holes at each end. Make a triangle using these sticks.

Make a rectangle using these sticks.

(If a child makes a square analyse the misconceptions/ learning difficulties and help him/ her to overcome the difficulty.)

Draw a circle on paper free hand.

Item 8:

The whole class can be posed with the following riddles on geometrical shapes and their properties that children have had some exposure to.

I have 4-equal sides, but my angles are not all equal. Who am I?

I have 3-sides. Two are equal. Who am I?

7.3 Visualisation and Spatial Reasoning

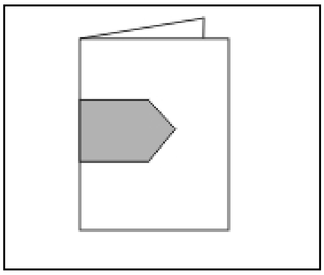
Starting from hands-on experience in playing with shapes at an early age, children develop mental representations of objects and also develop the skills to manipulate them mentally and visualise the results of enlarging, rotating, flipping, combining objects. They also develop the ability to predict what a shape would look like from different perspectives. Initially this is done by handling concrete objects. Later, visualisation without these aids would be possible.

Item 9

Show the child two identical cubes. Ask them if they were joined, what would be the shape of the shape of the objects and how many vertices would it have?

Item 10:

A sheet of paper was folded and a shape (as shown in the figure) marked on it.



If the shape were cut out from the folded paper, what would it look like when opened? (Students can be asked to draw it).

Item 11:

Give the child two right triangle (congruent) shaped cards. Ask her to use these two cards and make
(a) a square (b) a rectangle (c) a triangle.
When she has made each shape, ask her how she knows it is a 'square' or 'not a square. If she says a shape is not possible, you may ask her why it is not possible to make it?

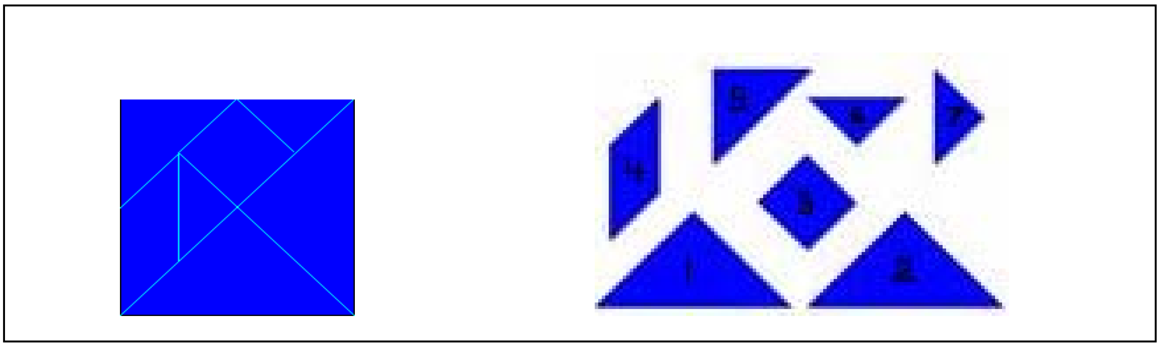
Later the same question could be asked where the child is expected to mentally manipulate the shapes.

Rama has two identical triangular tiles like these (two right triangles).
Is it possible to make a) a square b) a rectangle c) a triangle by joining them?
If possible, draw it. If not explain why it is not possible.

Item 12:

What shape will you get if you cut a rectangle along a diagonal?

Item 13:



i. A square using pieces 1 and 2.
ii. A triangle using pieces 1 and 2.
iii. A rectangle using pieces 3, 6 and 7.
iv. A triangle using pieces 4 and 5.
v. A triangle using pieces 4, 5 and 6.

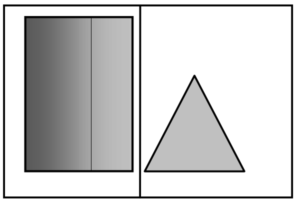
Older children can do similar and other exercises for shapes like parallelograms, trapeziums etc.

Item 14:

Show a solid shape, say a cylinder, to a child and ask what shape she would see if she were to look at it from top, or from different sides. This can be done with different solid shapes.

At a more advanced stage, the same can be done using a combination of solid shapes.

Item 15:



(a) (b)

Show drawings of a solid seen from two different positions.

Then show a cube, a cuboid, a triangular prism, a cylinder, a pyramid with a triangular base, etc. and ask 'Which of these objects would look like (a)? (b)? it be?'

How do you know that this is the shape it would look like?

Is this the only shape which it may appear to be?

Item 16:

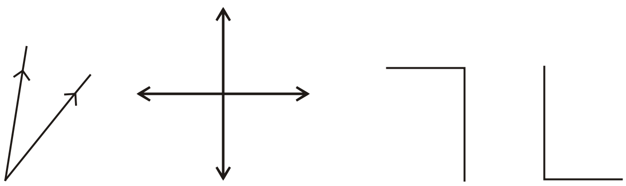
Provide the child with a set of unit cubes. Also give her drawings of top view and side view of objects and ask her to make shapes which look like each of the drawing.

7.4 Understanding fundamental geometric concepts

It is important for a student to understand geometric concepts and terms such as the idea of a straight line, parallel lines and angles at the primary level, to be able to solve more geometrical problems. Though these concepts are seemingly simple to understand, there can be several gaps in understanding. For instance, many children might not be able to visualise a right angle in an orientation where the arms are not vertical or horizontal. Students should understand angles not only as a measure, but also as a geometric shape and as movement (as in a rotation or sweep). For this reason, it is important to include tasks involving estimating, identifying, drawing and measuring angles in different orientations.

Item 17:

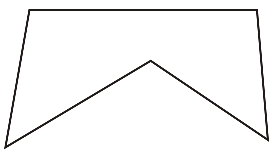
Which one of the following angles is not a right angle?



(a) (b) (c) (d)

Item 18:

Measure all the interior angles of the figure shown here.



Item 19:

Draw an obtuse angle with the given ray as one of its arms.

Item 20:

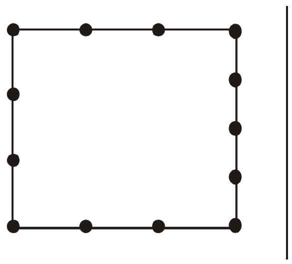
Materials: matchsticks and a partition.

A game can be played by two students or a teacher with a child.

The two children are on either side of the partition and each cannot see the other's work space. A shape is already made and provided to child 1. And he/she gives instructions to child 2 to reproduce the shape.

e.g. Child may say. Make a line 3 matchsticks long. At the end of the line, turn by 90 degree and go forward in a straight line by four steps. Turn again by 90 degrees and make a line parallel and equal in length to the first line. etc.

(Note the vocabulary used by child 1 and the ability of child 2 to follow descriptions/instructions).



Child 1 Child 2

Item 21:

Provide a set of three dimensional objects/two dimensional objects. Blindfold the child. Provide her/him with the set of objects and ask her to find among them various objects:

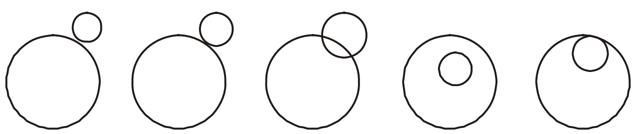
e.g. Find an object with one pair of parallel lines. Find an object with at least one pair of parallel lines. Find an object without any parallel lines, etc.

7.5 Understanding Important Geometrical Ideas

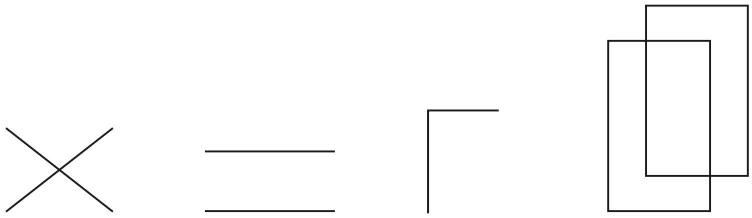
The primary level is also the age where important geometric ideas like that of congruence, symmetry and coordinates are developed. The students can be exposed to (and later assessed on) a variety of exploratory activities related to these ideas. Tasks using a dot grid or paper folding also lend themselves well for developing and assessing these ideas.

Item 22:

Provide the following drawings to the child and ask them to describe in words the relationship of the two objects to each other



(a) (b) (c) (d) (f)



(g) (h) (i) (j)

Item 23:

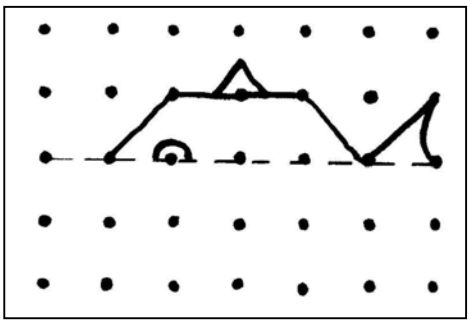
Give a collection of objects of 2-D shapes to the child. The collection should have some shapes that are congruent, some similar etc.
Task 1: Ask her to find a pair of objects that have exactly the same shape and the same size (congruent).
Task 2: Ask her to find different objects that are exactly the same shape but of different sizes (similar figures).

Item 24:

Provide two identical objects placed in specific relationship with each other and ask the child how you will get from one to the other.

Item 25:

Complete the following picture so that it is the same on both sides of the dotted line.



7.6 Developing Logical Thinking Abilities

Developing logical thinking is an integral part of the teaching of geometry and needs to be a part of every geometry classroom, including classes 1 and 2. This has to take place through tasks that involve making predictions, drawing conclusions and justifying the same using appropriate vocabulary. The tasks could pertain to any of the conceptual areas listed above.

For very young children, the tasks can be situated in the ordinary shape sorting and identifying tasks they do. Many of the investigative tasks listed in the sections on shapes and visualisation are also tasks based on logic and justification. The main thing is for the teacher to remember to ask more and more of ‘Why’ questions and a lot less of the usual ‘What’ and ‘Which’ questions in class.

Item 26:

Give a collection of shapes to students and ask them to sort them into two groups based on their own criterion. Ask them to justify the sorting criterion. While assessing, check whether the criterion is valid, and also whether the child is noticing different geometric attributes.

Item 27:

Students can be encouraged to make conjectures based on familiar shapes and terms related to them and informally prove or disprove them.

'Ali has some identical square tiles. He is trying to make a bigger square by placing them together (without overlapping). He claims that he has made a bigger square using 2 tiles. Can he? Why?'

8. Measurement

Measurement is an area of mathematics of immense practical importance. The concepts in measurement lie in doing and in relating to acts of measurement in the real world. Basically, measurement involves comparison of physical attributes of — an object: length, area, volume, time, weight, and angle are some of the basic attributes that are part of the learning of children in school.

The learning about measurement involves: the concepts, the related vocabulary and skills of using measuring instruments for measurement. The various dimensions of learning measurement are discussed below.

- Intuitive grasp of the attribute and concept of comparison and ordering
- Using an intermediary to compare two things (where direct comparison is not possible).
- Idea of a unit and iteration
- Standard unit and estimation
- Measurement of complex shapes involving combination of measurements
- Knowledge and use of instruments

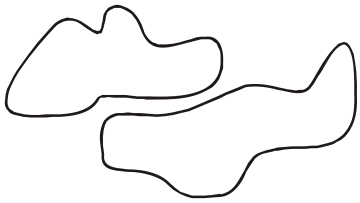
> Items to assess measurement focused on computation involving conversion of units. e.g.
> 3m 45cm + 4m 57cm = ?
> Convert 37.2kg to grams
> How many minutes are there in 16 hours?
>
> Such items tend to assess the child's competence and familiarity with computations involving decimals and related ideas rather than testing for conceptual development of measurement.

The present approach to assessment of concepts of measurement also tends towards using materials that need to be measured as well as measuring instruments in a problem solving mode.

8.1 Intuitive grasp of attribute and of comparison (ordering)

Item 1: (Intuitive sense of comparison and knowledge of comparative words)

Give the child two pencils, and ask: which one is longer and which one is shorter? Give two irregular flat shapes and ask, which is bigger? Observe how the child superimposes one over the other to compare area, whether she compensates for irregular bits that may stick out etc.



Provide hollow shapes and water/sand and ask to find out which has the larger volume/smaller volume.

For concept of weight two objects could be given and the child could be asked, which is heavier? Which is lighter?

Item 2: (comparison and techniques)

Give the child a set of pencils of different lengths. Pick one up from the set and ask: Find me a pencil longer than this one.
Find me a pencil shorter than this one.
Find me a pencil as long as this one.

Through this item, by observing the child's actions and answer one can tell if:

(a) The child has an intuitive understanding of the concept.
(b) The technique of comparison is in place or no — i.e. does she align the ends of the pencils, etc.

Item 3: (Comparison and Ordering)

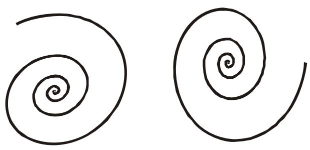
Give the child a set of pencils of different lengths and ask
Arrange these pencils from longest to shortest on the table/ground.

Observe how the child arranges these. Does she complete these by trial and error? Does she have a method in place to pick out and place the next one?

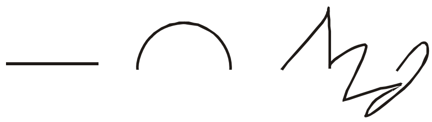
Give her another pencil, and now ask her to put it into the set. Observe how she identifies where the pencil should go in the series.

Item 4: (Concept for length of curvilinear objects)

Provide the child with two ropes, placed curled up and ask her which one is longer.



Provide her with a straight line and a curved line, and a zigzag line, all fixed, and ask her to tell you which one is longer. Additional materials, longer stick, shorter stick, rope (shorter and longer) may all be provided.



Similarly provide the child two objects and ask the child to measure the distance between them. Observe what part of the body she may use to make the measurement.

Item 5: (surface area of non-flat and solid figures)

Give the child two crinkled and bent pieces of paper and ask her which is larger.

Next give her a rectangular piece of paper, which is flat but flexible, and one which is stiff and bent into a curve like a part of a cylinder. Ask 'which is bigger?' and observe how she compares.

Provide her with two objects of same size—which could be pillow shaped. Both these pillows need covers. Now ask the child, 'which one will need more cloth?'

Item 6: (angles)

Ask the child to recognize various angles among the objects available in the room (chairs, tables, angles between blades of fan etc).

Children should be able to identify objects with edges and the 'angle' made at the joining of these edges.

Further they could be asked questions as to which angle is larger than the other. This would also give an idea if the child has a correct 'sense' of angle or has she confused it with distance or space between the two intersecting lines.

Ask children whether they can make acute/obtuse angles using fingers. One can start by spreading two fingers and then asking the child if she can see any angle there? What kind of angle is it?

You can position two fingers at a particular angle and ask the child what angle it is. You can follow this question with "if a person with longer fingers were to make the same position would the angle be larger?"

Through the above activities one can understand if the child has an intuitive understanding of the concept of angle. One can also clarify if the child has confused the concept of angle with a measure of length, area etc.

8.2 Using an intermediary to make comparison

Item 7:

Provide the child with two sticks of comparable length (but with a difference) and fixed at a distance from each other. Provide additional materials such as sticks, longer and shorter, rope, etc. Ask her to compare the two and tell you which is longer, and observe how she does it.

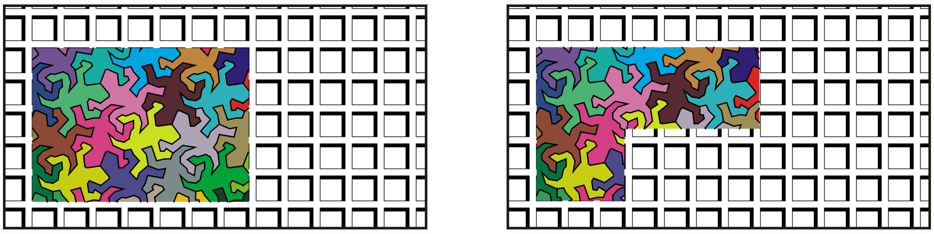
If she uses a longer stick, to make the measure and compare, next ask her to make the measurement using the shorter stick and whether she is able to iterate and count. This would tell us if the beginnings of the idea of a unit and iteration are forming.

The same item could be tried by providing the child with two very large tins or buckets and a small one with a much larger bucket holding water. Which of these two buckets will hold more water.

Item 8:

Provide the child with the picture alongside and ask, which carpet is larger?

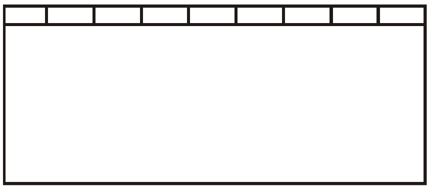
If the picture has a tiled floor (there could also be floor tiled with say triangular tiles and two carpets over it. By counting the number of tiles covered the child would have an estimate of the area.)



8.3 Iteration of units and inverse relation of unit size and measure:

Item 9:

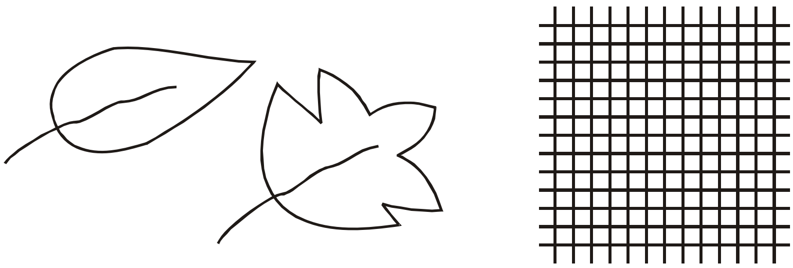
The length of the table measured in straws is about 7 straws. If I used matches to measure the length I would need ………… (more/less/as many)



Item 10:

Give the child two leaves of irregular shape and ask her to compare the areas and tell you which is larger. Along with other materials around such as rulers, string, also let there be square and triangular grid sheets.

Note if the child intuitively uses counting squares to compare the areas.



Item 11:

Provide shapes (regular/irregular/polygonal/circle etc) drawn on triangular grid and ask to compare areas (Which has the biggest area? Which has the smallest area?)

Shapes could also be drawn on a square grid for the same exercise.

Item 12:

Ask a child which takes longer: a ball being rolled down a slope or a ball being dropped from a height?

8.4 Standard measures and estimation

Item 13:

If the child has knowledge of standard units:

A meter is about as long as (a) the length of my arm

(b) the length of my book

(c) the length of my foot

Item 14:

Name an animal which is about 5 cm long.

Item 15:

Provide different objects to children and ask them to estimate their length.

Item 16:

How many square cm do you think will fit in at the top of the box: (draw a box about 5cm by 6cm) to understand if children have developed a sense of the size of the unit.

Similarly teacher may ask: how many meter square will cover the floor of this classroom.

Item 17:

Using standard weights to measure—children could be asked to weigh out one kilogram of rice/sand, and also 1.5 kilogram, 1 kilogram 100 grams etc. Later exercises could also include:

Find me an object about twice as heavy as this one, etc…

Item 18:

I have only 500 grams weight. How shall I measure one kilo? Two kilogram? (orally or practically)

Item 19:

How many weeks are there from 7 January to 24 February?

The summer holidays start on May 15th. Today is March 23. How many more days to go before the summer holidays start?

Item 20:

Materials:

Calendar for month of March.

Can you make the calendar for the month of April? On what day of the week will April 15th fall?

Item 21:

How long does it take to do the following things:

Eat dinner	
Blink	About twenty minutes
	Less than a second
Clap fifteen time	
Writing your full name	About 3 seconds

Item 22:

It takes about 4 hours by train from Bangalore to Mysore. If the train starts from Bangalore at 11am, at what time will it reach Mysore?

Item 23:

After measuring the length of the table, the carpenter wrote down 54 in his note book and forgot to mention the units. Which of these units do you think is the right one? Cm, m, or km?

Item 24:

Draw a triangle with sides 7cm, 5cm and 3cm.

Item 25:

Anita, Sunita and Vinita ran a race. Anita took 3 minutes and five seconds, Sunita took 4 minutes and 2 seconds, Vinita took 3 minutes and 7 seconds. Who won the race?

8.5 Measures of complex shapes and use of formulae

Item 26: (Perimeter and circumference)

Children tend to confuse these with area. Asking children to provide the perimeter of objects would tell us if they have grasped the concept.

Give two objects with same area but different perimeters (such as a rectangle of 16 × 4 and a square of 4 cm) and ask 'which has the longer perimeter?'

Item 27:

Present several objects, and ask which has the longest perimeter and shortest area (basically to understand if children are making out the difference between the two).

Item 28:

Ask the child to draw a square with a given perimeter, say 8 cm.

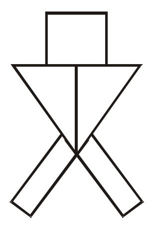
Ask the child to draw two different rectangles both with perimeter 20 cm.

Item 29: (Area of standard shapes and formulae)

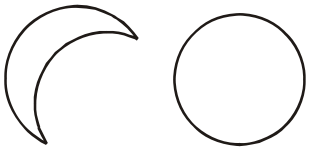
Provide square grid and ask the child to draw a square of length 4 cm on it, a rectangle of area 12 cm^2 on it, a triangle of area 12cm^2 on it.

Item 30: (Area of composite shapes as sum of parts)

Provide a complex shape made of sub-shapes and ask what is the total area of larger shape made of smaller ones.



Provide two shapes one concave and the other round and ask: which has the longer boundary.

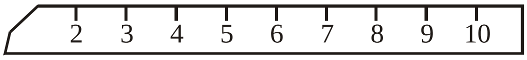


8.6 Knowledge of and use of instruments for measuring

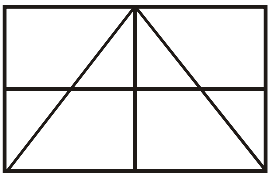
Rulers

Item 31:

Provide children with a strip of paper, and pencil, and ask them to make their own ruler. Observe what unit they choose, what markings they make, etc.



Item 32:



Provide children with a ruler with a broken edge (procure a ruler with zero mark broken or improvise a ruler) and ask them to measure with it.

Item 33:

Arrange a ruler with cm marking and ask what is the smallest length you can measure with this.

Item 34:

Give a 15 cm ruler and observe how the child measures object which is

(a) longer than 15 cm
(b) which is curved
(c) which is zigzag

Item 35:

Provide ruler and pencil and paper to the child and ask her to
(d) draw a line that is more than 12 cm long
(e) draw a line that is less than 5 cm long

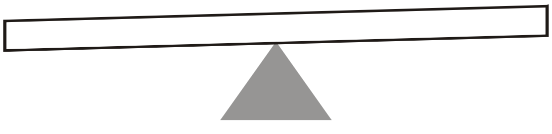
Item 36:

Place two pencils on either side of a cm marking ruler, (with bottoms not aligned) and ask to find out which pencil is longer.

Item 37:

Give a pencil to a child and ask: how long do you think this is?

Item 38: (Beam Balance)

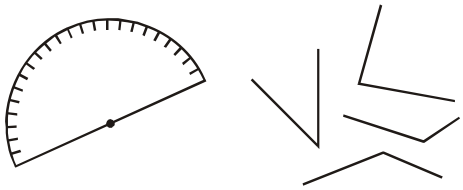


Give a beam balance to the child and 1kg weight. Provide an array of objects of different weights.

Ask her to sort out those that weight more than one kilogram, those that weigh almost one kilo, and those that weigh less than one kilogram.

Ask her to find a pair of objects which are equal in weight to each other/ give her one object and ask her to find another of the same weight.

Item 39: (Protractor)



Provide children with drawings of angles and a protractor. Ask her to measure the angles in the drawing.

The arms of the angle extend beyond the circumference of the protractor. Examine for whether the child is able to place the protractor for correct zero alignment and counts from zero.

The arms of the angle extend beyond the circumference of the protractor, but orientations are varied.

The arms of the angle may be shorter than the radius of the protractor and the child has to either extend arms or examine for alignment within.

Item 40:

Ask the child to construct a square. Examine whether the child uses a protractor or not, to construct right angles.

Item 41: (Clock)

Where will the hands of the clock be at 7 pm? At 8:30 pm, etc.

8.7 Indicators

The indicators/markers of quality for assessment of achievement in measurement for Classes I-V are presented in the box. The teachers can develop their own indicators based on the syllabus.

		Length
Class I	:	Distinguishes between near-far, thin-thick, shorter-longer/taller, low-high.
Class II	:	Measures lengths and distances along short and long paths using uniform (non-standard) units (for example, hand span, etc.)
Class III	:	Measures length using appropriate standard units-meters, centimeters.
Class IV	:	Determines area and parameter of simple geometrical figures.
	:	Solves problems involving length and distance.
Class V	:	Uses the four operations in solving problems related to length.
		Weight
Class I	:	Compares heavy and light objects.
Class II	:	Compares weights of two objects using simple balance.
Class III	:	Weights objects using standard units.
Class IV	:	Determines sums and differences of weights.
Class V	:	Uses the four operations in solving problems related to weight.
	:	Converts kilogram into grams.
		Time
Class I	:	Distinguishes between long and short duration of school days v/s holidays.
Class II	:	Reads the days of week and months of the year.
Class III	:	Reads a calendar to find a particular day and date
	:	Reads clock time correct to the hour.
Class IV	:	Calculates the number of weeks in a year.
	:	Reads clock time to the nearest hours and minutes.
Class V	:	Uses addition and subtraction in finding time intervals in simple cases.
		Money
Class I	:	Identifies common currency notes and coins upto Rs. 50/-
Class II	:	Puts together amount of money not exceeding Rs. 50/-
Class III	:	Converts rupees to paise
Class IV	:	Adds and subtracts amounts using column addition and subtraction with regrouping.
Class V	:	Uses the four operations in solving problems involving money.

9. Problem Solving, Patterns and Data Handling

It is necessary for problem solving tasks to be authentic. If children see the problem only as something belonging in school, and not anything that would interest an adult, a process of alienation sets in. On the other hand, an authentic problem, one where children see the benefit of solving it, brings satisfaction and hence enhances problem solving ability. For instance, when I ask a child to distribute 10 toffees among 10 children, the problem is not uninteresting. But it achieves authenticity when I ask further: "How are you sure that you have counted all the toffees and all the children? Is there any way to check?" This is because such verification is a real need for a child, and the ability to do so is immediately seen by the child to be rewarding. It is discussed in more detail in this section here we will suggest some aspects, which are more essential. Firstly, it is identifying, visualising and formulating the problem. Secondly, it is devising a strategy to solve it, and thirdly, it is verifying it. The ability to communicate, to support one's solution through argument, to follow an argument are also important facets developed in the problem solving approach.

Developing problem solving capabilities in children

It is important to provide children with ample opportunities to solve problems. Recreational mathematics often offers many such rich possibilities. Children may be exposed to a variety of techniques that would help in solving problem following the four steps–understand the problem, devise a plan to solve it, carry out the plan, check the result. (George Polya)

The central role that problem solving plays is best understood by focussing on students' needs in terms of solution strategies. Firstly, disposition is critical. If a student says, "I can't do it," He/she may be quite right in saying so. The disposition is what enables the student to attempt to solve the problem. Creating the right attitude and a positive environment in which to solve problems is important.

It is also important to acknowledge that there are problems of verifying difficulties. The implication is that from an early age, children should experience attempting problems whose solutions require different durations. Such variety goes a long way towards forming the mathematical predisposition referred to above. Perseverance is an important part of problem solving. We must dispel the myth that any problem that cannot be solved in a short duration is unsolvable.

Teachers need to use a range of suggestions that help students along, without actually offering the solution. Often, when a child is in trouble, she wants help to find the answer on her own, she does not want the answer itself. Asking, "Do you know the answer ?" is no help, as also telling the child to "read it again", or worse, an exhortation "to think a little harder".

If the clue is "LCM", saying "I know that you understand LCMs well, just look for one here" is more helpful than the question: "Don't you know your LCMs?"

One practical responsibility that the teacher has is to ensure space and time for all children to find solutions at their own pace. The key to doing this with an entire class is to make sure that no one shouts out the answer. This is one reason why it is important to design problems that offer a multiplicity of solutions.

9.1 Problem Solving

Although most people would agree that mathematics is about solving problems, it is not often clear what is meant by this, especially at the primary level. Mostly, it relates to the so-called "word problems", and in general, end-of-chapter exercises. But clearly there is much more to problem solving, that is linked to the child's skill and understanding a concept, and to his/her own confidence in formulating and solving problems.

- Identifying the problem and formulating the problem to be solved is the first important aspect of problem solving. As the above two heuristics suggest, finding out if the child has understood the problem, whether she is able to visualise and represent/model the problem.

- Then there is the planning of a strategy to solve, to be able to draw on existing knowledge to construct solutions, examine if they are valid, and evaluate possible solutions are all aspects to be assessed. Good problems are those that give students a chance to solidify and extend their knowledge and stimulate new learning. They would need to be meaningful, reasonable and authentic, and not contrived to draw upon some specific operation. A variety of strategies also may be employed to solve.

  Eventually, when students get used to the hints and clues offered, they start anticipating the clues, and that is when they employ heuristics. A heuristic is a plan of attack, which helps in approaching the problem in a particular way. A heuristic is designed to help solvers to approach, understand, and attempt the problem.

  A range of strategies are usually available for solving problems, such as using diagrams, looking for patterns, or trying special values or cases. These strategies need instructional attention if students are to learn them. Ideally, the curriculum should be structured so as to expose many problem solving strategies to children. Students also need to learn to monitor and adjust the strategies they are using as they solve a problem. Typical strategies appropriate for introduction at each level may be as follows, with the understanding that they are built cumulatively:

- the use of manipulatives, "act it out", draw a picture;
- guess and check, make a table, list or chart;
- "look for a pattern", use logical reasoning;
- create a model, estimate;
- the "work backwards" strategy.

During assessment one looks for the solution strategy itself and the ability of the child to use multiple skills.

- The ability to verify and interpret results and to generalise solutions represent a slightly higher stage of problem solving capabilities.
- Perhaps most important by, one would look for their attitude towards solving problems, their willingness to preserve, and their ability to attach any kind of problem in a systematic manner.

Frequently problems asked in textbooks are contrived, as the examples that follow illustrate.

If I had eight four-legged animals in my garden, how many legs are there in the garden ?

(Such exercises are worthwhile only as humour which is almost always lacking in mathematics texts).

A famous example asks: If 8 birds are sitting on a tree and one of them is shot by a gamester, how many remain on the tree? (The only sensible answer is that all survivors fly away immediately.)

Problems should be reasonable as well. A textbook asks: Mother made 152 puris and served as many of them as possible equally to all 12 members in the household. How many were left? Surely, with 152 puris, who cares? (And poor Mother!) Also, what kind of a household is it where each person consumes 12 puri's?

There are very few assessment instruments that are developed solely to measure student problem solving ability. Typically, problem solving is used to assess student ability to use knowledge in familiar situations or more recall the solution of a problem already solved by their or for them.

The teacher needs a range of problems for assessment of problem solving ability. Some tasks may be tough problems for some students, exercises for many, and mere recall for others. We must simply remember to challenge all students and provide repeated exposure to problem solving throughout the year. A positive attitude and repeated exposure is critical for success.

Assessment must take into account the difference in problem solving performance between formal "test" situations and informal classroom occasions. The former cause so much stress on the student that her approach may not be what comes naturally to her. Observation is a good assessment tool in the classroom, especially when students are working in groups. A teacher can best assess students' perseverance, self-confidence, interest, cooperation, contribution and much more, in such group environments.

Item 1:

The game is a role play and can be used to motivate students to solve the problem. The teacher or student can act as the storekeeper and give the list of items in the store and price for each item. For e.g. sugar Rs 20 per kg, rice Rs 18 per kg, oil Rs 50 per litre. The children are asked to act as customers and are given a list of items to buy by their parents (for example 5 kilos rice and 1 kilo sugar and 1 litre oil). They have to calculate the total amount of money they have to pay to the storekeepers.

Use this information to make up a problem.
e.g. Do I have enough money to buy one of each?
How many kilos of rice can I buy for Rs 75?

Item 2:

3→ 6   5→ 10, 10→ ?   What is my rule?

Item 3:

Material : Dices of different numbers on the faces.
Throw dices on the table. Read the number on the face. Aslk students to find the sum.

49	**50**	**26**	**25**

For example, add 49 and 50.
Ask them to suggest two ways of adding the numbers.

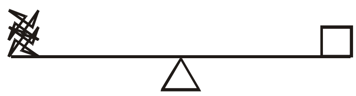
Ask them to carry out different operations and do each in at least two different ways.

Item 4:

I have six coins worth 60 paise. What coins do I have?

Item 5: (balance the beam)

Material : A blance, wooden block and weights of different denomination
Ask students to balance the beam.



9.2 Patterns

Mathematics curricula at school level includes the study of patterns. Invariably the curriculum designer desires children to look for patterns and thus enjoy the process of mathematical exploration. This translates to some number patterns and geometric patterns in textbooks, which do seem to interest children.

The development of abilities in the area of patterns include describing them, as requires comparing and contrasting, to observe similarity among differences, generalise (to go beyond the observed) verification and language for describing a perceived pattern instances to observe, similarity a language for describing a perceived pattern, sufficient engagement with the process to cause anticipation and finally, acknowledgment of the need for verification. These could well be listed as exploring, conjecturing, analysing, and proving processes that characterise mathematical thinking.

Several domains are accessible to children for exploration of patterns, beginning with counting, some of the earliest instances are those of skip counting: 1,3,5,7,… or 3,6,9,... Sequences also offer a lot of room for play: 1,2,3,7,11,… or 1,3,2,4,3,… , Some of these can be filled and extended in many ways. The calendar is a rich source of number patterns, and discoveries of relationships. ‘Primes’ are another very rich source, and has also fuelled great mathematics. Pattern explorations of shapes contribute richly to both the formal and informal learning of geometry.

Tasks are usually open ended or providing scope for going from one level to another and interaction with students to elicit their explanations or to alter and elaborate the task slightly to enrich their learning. Patterns could be varied for the complexity of the generative rules. They could also be patterns of numbers or of shapes.

Item 7:

Continue the series:

3,2,3,2,3,2…

3,3,3,2,3,3,3,2,…

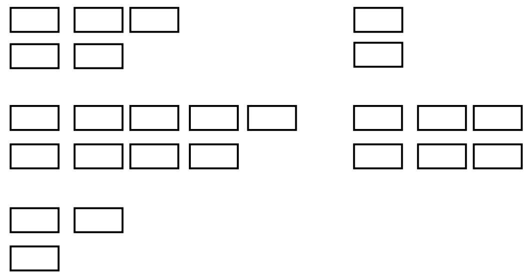
1,2,1,1,2,3,2,1,1,2,3,4,3,2,1,…

1,2,3,6,12,24,…

(a)

(b)

(c)



Item 8:

2 → 5

3 → 7

4 →

5 →

Item 9:

9,18,27,36,45,54,63,…

Find the relationship between the numbers.

Item 10:

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Find all the number patterns you can in this grid.

The same task can be given with number grids with other number of columns.

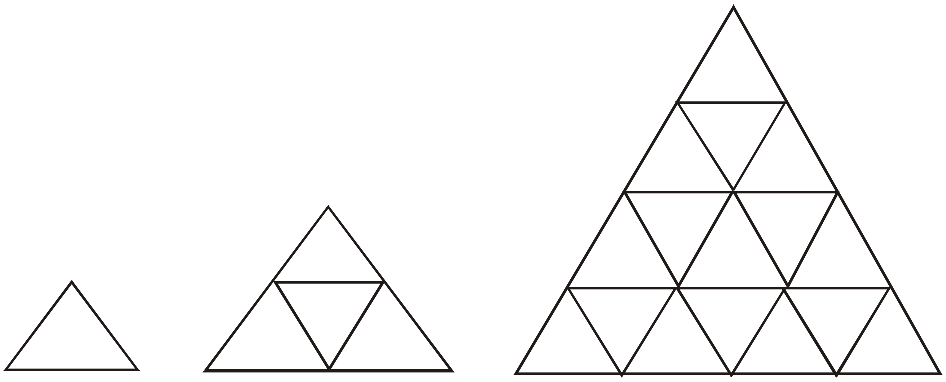
Students can also be asked to look and locate specific patterns. E.g. in the above number grid, find the tables of four.

Students can also be asked to make number grids that satisfy some specified conditions. E.g. Make a grid where the tables of six come in the last column.

E.g. Make a number grid where the tables of thirteen appear in along a diagonal.

Item 11:

Provide a large set of small size equilateral triangles, some of which are organised into the pattern below:



How many triangles do you need next?

Make the next triangle.

9.3 Data Handling

An ability to collect, classify and interpret data is considered an important aspect of mathematical literacy. Increasingly, use of tables, diagrams and questionnaires is an integral part of everyday life, and this is a skill required not only for reading newspapers but often, even textbook material. Moreover, graphical forms and tables are a critical tool for communication, and enrich the child's toolkit for formal presentation of information. Later on, when the child is called on to critically judge presented data (which is arguably essential for a literate person), one who has not been trained in methods of presentation is likely to be easily manipulated.

Typically data interpretation and analysis is introduced in the upper primary stage and strengthened in the secondary stage of the ten-year school. However, increasingly, in many countries there is a move towards introducing data handling at the primary stage, and this is a view endorsed by the NCF 2005 as well. Important reasons for this include making connections with other branches of study early on (especially language and social studies), as well as relate arithmetic to everyday life situations. A more conceptual reason is that data analysis exposes children to uncertainty and changeability in information, as opposed to the certain and fixed nature of mathematical facts they mostly study.

However, at the primary level, the emphasis is mainly on exposure to presentations of data, and intuitive understanding (at the concretely experienced level) rather than any formal statistics, nor or the children expected to come up with their own presentations. Children are encouraged to comprehend how items of quantitative information relate to one another, and learn to make use of formal means, especially in the form of tables.

The following represent some of the key areas of data handling which form the basis of children’s experiences and which may be assessed:

- Reading data represented in different forms and simple interpretation, beginning with answering questions based on the information in the representation, and understanding what questions can and cannot be answered with the data provided.
- More complex interpretations and inferences from represented data, hypotheses and verification based on the data.
- In problem solving, understanding the nature of data to be gathered and later handling it.
- Representing data in the form of tables, pictograms, simple histograms and pie charts, (using suitable units etc.).

Thus using information that has already been processed and represented in the form of tables and charts, and beginning with situations and problems which require data to be gathered and represented data to be interpreted are among the natural situations in the classroom that offer themselves for exercises during which assessment of the development of children’s capabilities can be undertaken. The subject matter of environmental studies and social studies also offers many opportunities for gathering and handling data.

Note that the objective of such assessment is mainly to see how children prise out relevant quantitative data, how they see the numbers involved possess meaning in context, perform operations on them and apply the result back into the original context to construct new meaning. Moreover, children's understanding of shapes and visual reasoning is called upon in data analysis.

Close attention is to be paid to whether children are overly tied to particular modes of presentation, by presenting the same information in different ways. Moreover, data handling exercises present an opportunity to discriminate whether a child's difficulty is due to linguistic incomprehension, lack of understanding of arithmetical operations, or lack of fluency with numbers.

Illustrative Items:

Item 12:

The class time table has a complex structure, and analysing it opens up ways of unlocking children's understanding. An attempt to answer the following queries offers much scope to see how children understand tables per se:
How many hours per week are spent in mathematics ?
Which is the activity that takes up the least time over a week ?
Which are the subjects studied daily ? alternate days ?
Can play time be doubled, but without reducing time for any subject by more than one hour ?

Similarly, simple tables of bus / train timings of a town can be subject to questioning.

Item 13:

A simple schematic of a bus route (preferably the school bus, if one is used) offers room for questions like:
How many stops does it make ?
How many stops between X and Y ? Which is the third stop after X ? etc.

Item 14:

Children can together construct a detailed schematic map of the school, a road, a village. Here, the critical idea is not scale, but relative distances.
Which is nearer from the HM's room — the coconut tree or the well?
How far is the toilet from one's classroom? etc.

Item 15:

Collecting statistics (within the classroom) of children's height, weight, favourite colours / foods / games, number of siblings, even length of nose or index finger, can be a source of great entertainment and education, as well as an opportunity for assessment of data handling. In particular, access to information by indexing can be assessed.
How many prefer red to blue ?
Which is the colour most preferred among girls?
The use of graphics for visual presentation of such data is welcome, mainly to see how much use children can make of such forms.

Item 16:

Before giving this activity teacher gives the table of height and weight, which shows what height, should have what weight.
The table is an example and it is not prescribed by doctor.

Height	Ideal weight	Normal weight	Over weight	Undeweight
5 feet	50-55 kgs			
5’ 1”	56 – 60 kgs			
etc	etc			

The teacher can use the above chart (the ideal weights given are just for illustration)

Ask a child to use the weighing scale and weigh 10 other classmates, and use a height scale on a wall to measure their heights.

Ask the child to classify the children into normal, over weight, under weight.

Observe how the child goes about collecting information.

Observe the way the child collects the data – does she use a paper, pencil and a format in which to capture the data..

See if the child is able to classify each child into the categories – observe the approach to approximation, classification.

Observe the child’s approach to presenting her findings; if she is able to illustrate the findings well.

Item 17:

On a large chart, the teacher (or older children) marks twelve columns, corresponding to the months of the year. Each child is asked to bring an empty matchbox from home, and to mark it in any way. (In one school, white paper was stuck on each match box, and children drew patterns of their choice on the paper.) Each child pastes the match box in the column corresponding to the month of her birth.

The teacher can prod the children into thinking about questions like..

- In which month do we have the most birthdays?
- In which month do we have the least birthdays?

-How many birthdays fall in the first three months of the year?

Here children are analyzing and interpreting the data and teacher can assess these skills by asking the above questions to the individual child.

Item 18:

The teacher cuts out thin long strips of paper and hands them to the children. Each child measures the circumferencef her/his head with it, making a mark on the paper to show this length and cuts it. All the children write their names on their strips and paste them on a long board, using the same base line.

The teacher could begin the process of getting children to analyse the data, — whose heads are smaller than whose?

- How many children have same size of head? etc..
- How many heads are bigger then you?

By asking the above questions the teacher can assess whether the child can classify, relate and differentiate between the measurement of the circumference of his head with other.

10. Recording and Reporting

The recording of the progress of the achievement of children in mathematics has drawn our attention. The assessment of children's progress is being considered continuous process. Unlike the earlier approach of right and wrong in awarding marks to the students, the present approach is process based. A teacher has more responsibility towards children in process based assessment. One has to carefully observe the children's work. Any problem in arithmetic is to be divided into small steps. Each step involves a process. A child needs to be recognised and credited for each process successfully completed. Based on this approach, each problem is to be assessed and categorised into three grades or levels.

Level 1/ Grade 1: A child cannot successfully complete even a part of the process.
Level 2/ Grade 2: A child can partly complete the entire process
Level 3/ Grade 3: A child can successfully complete the entire process.

Keeping in mind grass root reality a teacher, after completing a topic, or even during the teaching learning process may observe the students for the steps involved in a problem. Thus at a time a teacher needs to keep the record of 4/5 students. An example of a format for recording is presented below. The teacher many modify the format according to the requirement of the system and **consensus arrived** in consultation with other teachers and the headmaster.

Format of recording for a child (Grade III)

S. No.	Topic	First Quarter			Second Quarter			Third Quarter		
		Class Work	Assig nmen t	Practica l	Class work	Assignme nt	Practica l	Class work	Assig nment	Practica l
1.	Shapes & Design									
2.	Number reading & writing									
3.	Addition									
4.	Subtraction									
5.	Time									
6.	Calender									
7.	Weight									
8.	Money									
9.	Volume									
10.	Multiplication									
11.	Division									
12.	Data H andling									

A teacher may fill the grade level in the blank space. The information thus recorded may be shared with the parents in positive words. The parents are to be informed the potentialities of the child and that the child has successfully completed the following process in arithmetic.

The process of learning mathematics consists of observation, discussion, expression, translation, explanation, classification, questioning, analysis, synthesis experimentation, categorisation, measurement and calculation. These processess have many steps. For example, the process of discussion involves steps of listening, talking, expressing opinion, finding, reading etc. A person with a thorough knowledge of the content of the arithmetic may assess each one of them in three grades. In that case the topic in the above format may be replaced by process. In the context of mathematics a process is always coupled with content of arithmetic. Therefore teachers may take a note of this reality and nature of the subject.